

Solution

Class 12 - Physics

2020-2021 - Paper-3

Section A

- a. At the two magnetic poles, the magnetic needle rests vertically. Hence, at poles, $\delta = 90^\circ$
Thus, Horizontal component, $B_H = B \cos \delta = 0^\circ$

b. At a place on the magnetic poles, earth's magnetic field is perpendicular to the surface of earth, i.e., vertical.
Therefore, angle of dip (δ) = 90°
- When the magnetic moment of the magnetic dipole is antiparallel to the magnetic field ($\theta = 180^\circ$), its potential energy is maximum.
- Since magnetic flux increases when the loop moves into uniform magnetic field. So according to lenz law, current will be in the clockwise direction.
- The total number of magnetic lines of force crossing the surface A in a magnetic field \vec{B} is termed as magnetic flux.
 $\phi = BA \cos \theta$
Its SI unit is Weber. It is a scalar quantity.

OR

According to Lenz law the induced emf always opposes it's cause. Yes, emf will be induced in the rod, as there is change in magnetic flux linked with the rod due to earth's magnetic field.

- Resistance R is less for copper loop, so the current induced is larger in the copper loop.

OR

First law: Whenever a conductor is placed in the varying magnetic field, electromagnetic fields are induced known as induced emf. If the circuit is closed, a current is also induced which are called induced current.

2nd Law: It states that the induced emf in a coil is equal to the rate of change of flux. Here the flux is the product of the number of turns in the coil and flux connected with the coil. The formula of Faraday's law is given below:

$$\varepsilon = -N \frac{d\phi}{dt}$$

- Here, $M = 1 \text{ mH} = 1 \times 10^{-3} \text{ H}$

and $I = 0.5 \text{ A}$

$$\therefore \phi = MI = 1 \times 10^{-3} \times 0.5 = 5 \times 10^{-4} \text{ Wb}$$

OR

One ampere current is the current which flows through each of two parallel uniform linear conductors of length 1m each, which are placed in free space at a distance of 1m from each other and for which they attract or repel each other with a force of $2 \times 10^{-7} \text{ N}$ in that space.

- This is because magnetic lines of force are continuous closed loops and mono pole is not possible.

- As $F = qvB \sin \theta$

So, an electron moving through a magnetic field does not experience any force when either electron is moving parallel to the direction of the magnetic field or it is at rest.

- Area = a^2 , And current = I

The magnetic moment, associated with the coil is

$$NIA = 1 \times I \times a^2$$

$$\vec{\mu}_m = Ia^2 \hat{i} \text{ along x-axis}$$

- When a block of metal is kept in a varying magnetic field then Eddy current is produced.

- (d) A is false and R is also false

Explanation: In the case of the metallic rod, the charge carriers flow through the whole of the cross-section. So, the magnetic field exists both insides as well as outside. However, the magnetic field inside the rod will go on decreasing as we go towards the axis.

12. (d) A is false and R is also false

$$\text{Explanation: } r = \frac{mv}{qB} = \frac{\sqrt{2m(\text{KE})}}{qB}$$

When KE is halved, the radius is reduced to $\frac{1}{\sqrt{2}}$ times its initial value.

13. (a) Both A and R are true and R is the correct explanation of A

Explanation: The magnetic field used in a moving coil galvanometer is very strong. The earth's magnetic field is quite weak as compared to the magnetic field produced by the field magnet. Practically the coil rotates under the effect of the strong magnetic field due to the field magnet and the weak magnetic field due to the earth does not affect the working of the moving coil galvanometer.

14. It is clear that North pole of the magnet is moving away from coil PQ, so current will be induced in such a way that it acts as South pole. Hence, the direction of current will be anti-clockwise. Again, the South pole is approaching towards coil CD, so end C of the coil will act as South pole. Hence, the direction of current will be clockwise.

Section B

15. (d) 136 V

$$\text{Explanation: } e = NBA\omega = NBA2\pi n = 8 \times 0.5 \times 0.09 \times 2 \times 3.14 \times 60 = 136\text{V}$$

16. (c) Power

Explanation: Energy losses be zero in transformers hence power remains constant in step down and step up transformer also.

17. (a) $\frac{M}{2}$

Explanation: Since magnetic moment is given by product of pole strength and length of dipole, when it is cut into two pieces of half the length, each piece will have magnetic moment equal to half of the original piece.

18. (c) material of the turns of the coil

Explanation: For a coil

- A current-carrying circular coil behaves like a bar magnet whose magnetic moment is $M = NiA$; where N = number of turns in the coil, i = current through the coil and A = area of the coil.

It depends upon N , I and A . But it does not depend upon the strength of the magnetic field.

19. (c) 2 : 1

Explanation: A solenoid is equivalent to a bar magnet.

For points at distances greater than the length of the solenoid, the field along the axis of the solenoid is

$$B_{axial} = \frac{\mu_0 2m}{4\pi x^3} \text{ and along the perpendicular bisector or equatorial line is}$$

$$B_{equatorial} = \frac{\mu_0 m}{4\pi x^3}$$

$$\text{Therefore, } \frac{B_{axial}}{B_{equatorial}} = \frac{2}{1}$$

20. (c) iLB

Explanation: Magnitude of the Force experienced by a current carrying conductor placed in a magnetic field is $ILB \sin \theta$.

If the angle between the directions of the current and the magnetic field is 90° , $F = iLB$

21. (a) $B \perp v$

Explanation: The Biot-Savart law states how the value of the magnetic field at a specific point in space from one short segment of current-carrying conductor depends on each factor that influences the field. The magnitude of \vec{B} is $B \propto |q|$; $B \propto v$; $B \propto \sin \phi$, $B \propto \frac{1}{r^2}$

$$B \propto \frac{|q|v \sin \phi}{r^2}$$

$$B = \frac{\mu_0 |q|v \sin \phi}{4\pi r^2}$$

where $\frac{\mu_0}{4\pi}$ is a proportionality constant, 'r' is the magnitude of position vector from charge to that point at which we have to find the magnetic field and ϕ is the angle between \vec{v} and \vec{r} .

$$\text{or } \vec{B} = \frac{\mu_0}{4\pi} \frac{|q|(\vec{v} \times \vec{r})}{|r^3|} \hat{n}$$

Where \hat{n} is the direction of \vec{B} which is in the direction of cross product of \vec{v} and \vec{r} . Or we can say that $\vec{B} \perp$ to both \vec{v} and \vec{r} .

where is a proportionality constant, V is the magnitude of position vector from charge to that point at which we have to find the magnetic field and ϕ is the angle between v and F .

Where h is the direction of B which is in the direction of cross product of v and F . Or we can say that $\vec{B} \perp$ to both v and F .

22. (a) Zero

Explanation: The magnetic field of a toroid is limited in the annular region of the toroid. It is zero in the empty region enclosed by it and also outside it.

Section C

23. For the undeflected motion of the particle, total force on the particle is zero. So,

Force due to electric field + Force due to the magnetic field = 0

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

$$\Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$$

From the figure, $\vec{v} = -v \hat{i}$ and $\vec{B} = -B \hat{k}$

As the particle is moving along X-direction and the magnetic field is perpendicular to the plane of the paper directed inwards, i.e. Z-direction

$$\text{Thus, } \vec{E} = -(-v \hat{i} \times -B \hat{k}) = -vB(-\hat{j}) = vB \hat{j}$$

Therefore, Magnitude of electric field is vB and direction of electric field will be perpendicular to both v and B , i.e. along positive Y-axis.

24. Magnetic field due circular wire P,

$$B_P = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \text{ (along vertically upwards)}$$

Magnetic field due to circular wire Q,

$$B_Q = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \text{ (along horizontal towards left)}$$

Net magnetic field at the common centre of the two coils,

$$B = \sqrt{B_P^2 + B_Q^2} = \sqrt{2} \times \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \text{ (}\because B_P = B_Q\text{)}$$

As the fields produced by the two coils is equal (in magnitude), the net magnetic field will be inclined equally to both the coils.

25. Given, $I_1 = 300 \text{ A}$, $I_2 = 300 \text{ A}$, $r = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$

$$\text{By using formula } F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

$$\text{Or force per unit length} = \frac{F}{l} = \frac{4\pi \times 10^{-7} \times 300 \times 300}{2\pi \times 0.015} = 1.2 \text{ Nm}^{-1}$$

The total force between the wires is

$$F = fl$$

$$= 1.2 \times 0.70 \text{ N} = 0.84 \text{ N}$$

The force is repulsive since the current will flow in opposite direction in the two wires connecting the battery to the starting motor.

OR

Equivalent magnetic moment of the coil,

$$\vec{M} = IA\hat{n}$$

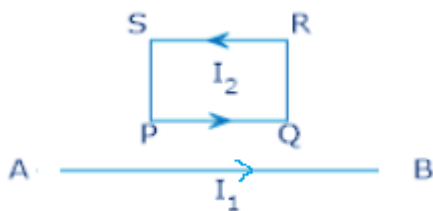
$$\therefore \vec{M} = Ilb\hat{n} \text{ (}\hat{n} \text{ = unit vector } \perp \text{ to the plane of the coil), Area is given by } lb$$

$$\therefore \text{Torque} = \vec{M} \times \vec{B}$$

$$= Ilb\hat{n} \times \vec{B}$$

$$= 0 \text{ (as } \hat{n} \text{ and } \vec{B} \text{ are parallel or antiparallel to each other)}$$

26.



Since current in arm PQ and in wire AB are in same direction, therefore wire AB will attract the arm PQ with a force (say F_1). But wire AB repels the arm RS with a force (say F_2) as the currents are in opposite directions. Now, since arm PQ is closer to the wire AB, $F_1 > F_2$ i.e. the loop will move towards the wire.

27. i. The magnetic field of earth at any place on it can be completely described by three parameters which are called elements of magnetic field of earth. They are as follows:
- Angle of declination (α): Angle between magnetic meridian and geographic meridian at any place.
 - Angle of dip (δ) or magnetic inclination: Angle between total magnetic field of earth and horizontal direction in magnetic meridian at any place.
 - Horizontal component of earth's magnetic field (B_H): Component of earth's magnetic field in horizontal direction in magnetic meridian.

ii. At the magnetic equator, the needle shows horizontal direction, so the angle of dip is zero at the magnetic equator.

28. Magnetic Moment of the bar magnet, $M = 0.32 \text{ JT}^{-1}$

External magnetic field, $B = 0.15 \text{ T}$

- i. For the system to be in stable equilibrium the angle θ , between the bar magnet and the magnetic field is taken as 0° .

$$\text{Potential energy of the system} = -MB \cos \theta$$

$$= -0.32 \times 0.15 \cos 0^\circ$$

$$= -4.8 \times 10^{-2} \text{ J}$$

- ii. For unstable equilibrium, the bar magnet is oriented at 180° to the magnetic field.

$$\theta = 180^\circ$$

$$\text{Potential energy} = -MB \cos \theta$$

$$= -0.32 \times 0.15 \cos 180^\circ$$

$$= 4.8 \times 10^{-2} \text{ J}$$

OR

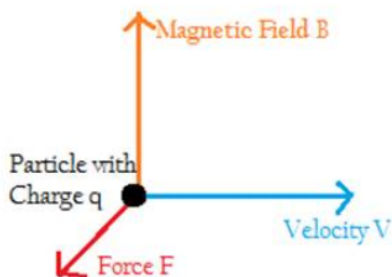
When a charged particle q moves with a velocity v in a magnetic field B , enters a magnetic field it experiences a force known as Magnetic Lorentz Force which is given by $\vec{F} = q(\vec{v} \times \vec{B})$

$$|\vec{F}| = (q|\vec{v}||\vec{B}|\sin \theta)\hat{w}$$

Now, due to magnetic Lorentz force, the particle accelerates so the velocity of particle changes, Magnetic field changes in both magnitude and direction so the Magnitude of Force on particle also keeps on changing, hence the velocity of the particle also changes.

In this Case force on the particle is always perpendicular to its velocity, which only changes the direction of the particle and its speed remains same, i.e. velocity of particle changes due to change in direction of particle and magnitude of velocity or speed same as it entered.

The following figure depicts the direction of the force on the particle at any instant:



29. The iron bar must be a magnet. When the bar magnet falls through the copper shell, it produces a change in magnetic flux through it. Due to this, eddy currents are produced in the shell. The eddy currents so produced

oppose the fall of the magnet (Lenz's law) and hence bar magnet experiences retarding force.

30. Magnitude of Induced emf is directly proportional to the rate of area moving out of the field for a constant magnetic field.

$$\varepsilon = -\frac{d\phi}{dt} = -B\frac{dA}{dt}$$

For the rectangular coil, the rate of area moving out of the field remain same while it is not so for the circular coil. Therefore, the induced emf for the rectangular coil remains constant.

31. i. The primary coil P should be moved faster towards the secondary coil S.

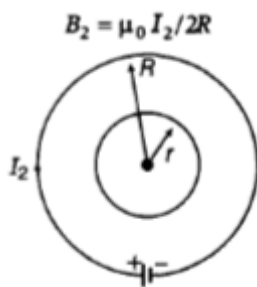
ii. The related law is Faraday's laws of electromagnetic induction.

The results of Faraday's experiments on electromagnetic induction are known as Faraday's laws of electromagnetic induction. These laws are stated as below:

- Whenever magnetic flux linked with a circuit (a loop of wire or a coil or an electric circuit in general) changes, induced e.m.f. is produced.
- The induced e.m.f. lasts as long as the change in the magnetic flux continues.
- The magnitude of the induced e.m.f. is directly proportional to the rate of change of the magnetic flux linked with the circuit.

OR

Let a current I_2 flows through the outer circular coil of radius R . The magnetic field at the centre of the coil is



As $r \ll R$, hence field B_2 may be considered to be constant over the entire cross-sectional area of inner coil of radius r . Hence, magnetic flux linked with the smaller coil will be

$$\phi_1 = B_2 A_1 = \frac{\mu_0 I_2}{2R} \cdot \pi r^2$$

$$\text{As, by definition } \phi_1 = M_{12} I_2$$

$$\text{Now mutual inductance, } M_{12} = \phi_1 / I_2 = \frac{\mu_0 \pi r^2}{2R}$$

$$\text{Say } M_{12} = M_{21} = M,$$

$$\therefore M = \frac{\mu_0 \pi r^2}{2R}$$

Section D

32. To find the charge that passes through the circuit first we have to find the relation between instantaneous current and instantaneous magnetic flux linked with it. The emf induced can be obtained by differentiating the expression of magnetic flux linked w.r.t. t and then applying Ohm's law, we get A rectangular loop of wire ABCD is kept close to an infinitely long wire carrying a current

$$I = \frac{E}{R} = \frac{1}{R} \frac{d\phi}{dt}$$

According to the problem electric current is given as a function of time.

$$I(t) = \frac{dQ}{dt} \text{ or } \frac{dQ}{dt} = \frac{1}{R} \frac{d\phi}{dt}$$

Integrating the variable separately in the form of the differential equation for finding the charge Q that passed in time t , we have

$$Q(t_1) - Q(t_2) = \frac{1}{R} [\phi(t_1) - \phi(t_2)]$$

$$\phi(t_1) = L_1 \frac{\mu_0}{2\pi} \int_x^{L_2+x} \frac{dx'}{x'} I(t_1) [\phi_m = \vec{B} \cdot \vec{A} = \frac{\mu_0 I}{2\pi} l \int_{x_0}^x \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln \frac{x}{x_0}]$$

$$= \frac{\mu_0 L_1}{2\pi} I(t_1) \ln \frac{L_2+x}{x}$$

Therefore the magnitude of charge is

$$Q = \frac{1}{R} [\phi(T) - \phi(0)]$$

$$Q = \frac{\mu_0 L_1}{2\pi} \ln \frac{L_2+x}{x} [I(T) - I(0)]$$

Now $I(T) = I_1$ and $I(0) = 0$

$$\therefore Q = \frac{\mu_0 L_1}{2\pi} I_1 \ln\left(\frac{L_2+x}{x}\right)$$

This is the required expression.

33. i. Low retentivity

High permeability

ii. Gauss' Law for magnetism applies to the magnetic flux through a closed surface. In this case the area vector points out from the surface. Because magnetic field lines are continuous loops, all closed surfaces have as many magnetic field lines going in as coming out. Hence net magnetic flux through any closed surface is zero.

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

Gauss law in electrostatics.

$$\oint \mathbf{E} \cdot d\mathbf{s} = \frac{q}{\epsilon_0} \text{ i.e., net electric flux through any closed surface is } \frac{1}{\epsilon_0} \text{ times charged enclosed by the surface.}$$

The Gauss law in magnetism indicates that monopole does not exist.

OR

a. Since, $\tau = MB \sin \theta$, here M is the magnetic moment and B is the magnetic field, hence

$$\therefore M = \frac{\tau}{B \sin \theta}$$

$$\text{or } M = \frac{0.016}{800 \times 10^{-4} \times \sin 30^\circ}$$

$$\text{or } M = 0.40 \text{ Am}^2 \text{ [}\therefore 1\text{G} = 10^{-4}\text{T]}]$$

b. $W = -MB [\cos \theta_2 - \cos \theta_1]$

$$W = -MB (\cos 180^\circ - \cos 0^\circ) = -MB(-1 - 1) = 2MB$$

$$W = 2 \times 0.40 \times 800 \times 10^{-4} = 0.064 \text{ J}$$

c. $m_s = NIA$. From part (a), $m_s = 0.40 \text{ Am}^2$

$$0.40 = 1000 \times I \times 2 \times 10^{-4}$$

$$I = \frac{0.40 \times 10^4}{(1000 \times 2)} = 2 \text{ A}$$

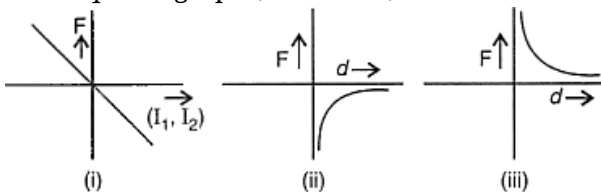
34. The magnitude F of the force between two straight parallel current-carrying conductors kept at a distance d apart in the air. We know that F is an attractive (-ve) force when the currents I_1 and I_2 are 'like' currents, i.e., when the product $I_1 I_2$ is positive or when they are flowing in the same direction.

Similarly, F is a repulsive (+ve) force when the currents I_1 and I_2 are 'unlike' currents, i.e., when the product $I_1 I_2$ is negative or when they are flowing in the opposite direction.

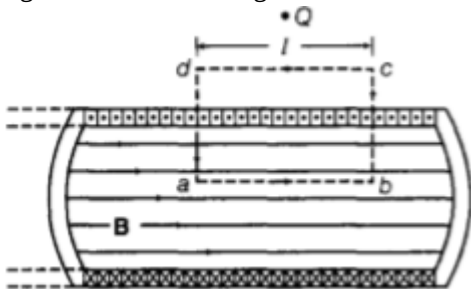
Now $F \propto (I_1 I_2)$, when d is kept constant and $F \propto \frac{1}{d}$

when $I_1 I_2$ is kept constant.

The required graphs, therefore, have the forms shown below.



35. Figure shows the longitudinal sectional view of current carrying long solenoid.



Let B be the magnetic field at any point inside the solenoid along ab. The current comes out of the plane of paper at points marked.

The rectangular Amperian loop abcd is considered to determine the magnetic field.

Applying Ampere's Circuital Law over loop abcd,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times (\text{Total enclosed current})$$

$$\int_a^b B \cdot dl + \int_b^c B \cdot dl + \int_c^d B \cdot dl + \int_d^a B \cdot dl = \mu_0 \left(\frac{N}{L} l I \right)$$

where, $\frac{N}{L} = n$, number of turns per unit length and $ab = cd = l =$ length of rectangular Amperian loop, and total number of turns = $\frac{N}{L} l$

As the field outside the solenoid is zero, $\int_c^d B \cdot dl = 0$,

$$\int_a^b B dl \cos 0^\circ + \int_b^c B dl \cos 90^\circ + 0 + \int_d^a B dl \cos 90^\circ = \mu_0 \left(\frac{N}{L} \right) l I$$

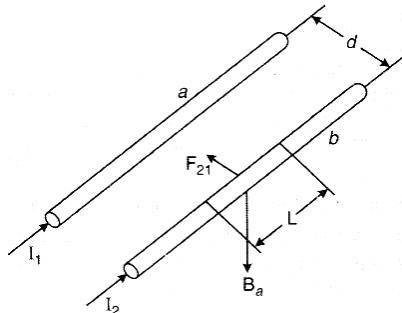
$$B \int_a^b dl = \mu_0 \left(\frac{N}{L} \right) l I \Rightarrow B l = \mu_0 \left(\frac{N}{L} \right) l I$$

$$\Rightarrow B = \mu_0 \left(\frac{N}{L} \right) I$$

$$\text{or } B = \mu_0 n I$$

This is a required expression for magnetic field inside the current carrying long solenoid.

OR



B_a = magnetic field due to wire 'a'

B_b = magnetic field due to wire 'b'

The magnetic field, due to wire 1, at any point on the wire 2, is directed normal to the direction of current flow in wire 2.

Magnetic field around wire 2 due to a current I_1 in wire

$$B_a = \frac{\mu_0 I_1}{2\pi r}$$

Force upon conductor carrying current due to magnetic field

$$\vec{F} = I(\vec{l} \times \vec{B})$$

$$\therefore \text{Force, } F_{21} \text{ on a length } l \text{ of wire 2} = I_2 l \frac{\mu_0 I_1}{2\pi r}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi r} \cdot l$$

$$\text{Similarly, } F_{12} = \frac{\mu_0 I_1 I_2}{2\pi r} \cdot l$$

The nature of the force is repulsive for currents in opposite direction and attractive when currents flow in the same direction.

36. Here, $M = 1.6 \text{ Am}^2$, $d_1 = 20 \text{ cm}$, $H = ?$

When north pole of magnet is pointing south, neutral points lie on axial line of magnet. At neutral point,

$$B_{ax} = \frac{\mu_0}{4\pi} \frac{2M}{d_1^3} = H$$

$$\text{or } H = \frac{10^{-7} \times 2 \times 1.6}{\left(\frac{1}{5}\right)^3} = 4 \times 10^{-5} \text{ tesla}$$

When magnet is reversed i.e. north pole is pointing north, neutral points lie on equatorial line of the magnet.

We have to calculate d_2 .

$$\text{As } B_{eq} = \frac{\mu_0}{4\pi} \frac{M}{d_2^3} = H$$

$$\therefore d_2^3 = \frac{\mu_0}{4\pi} \frac{M}{H}$$

$$d_2^3 = \frac{10^{-7} \times 1.6}{4 \times 10^{-5}} = 0.4 \times 10^{-2} \text{ m}^3$$

$$= 0.4 \times 10^{-2} \times 10^6 \text{ cm}^3$$

$$= 4000 \text{ cm}^3$$

$$\text{Thus, } d_2 = (4000)^{1/3} = 15.87 \text{ cm}$$

Section E

37. a. $B = \frac{\mu_0 NI}{2R}$

Here, $N = 100$; $I = 3.2$ A, and $R = 0.1$ m. Hence,

$$B = \frac{4\pi \times 10^{-7} \times 10^2 \times 3.2}{2 \times 10^{-1}} = \frac{4 \times 10^{-5} \times 10}{2 \times 10^{-1}} \text{ (using } \pi \times 3.2 = 10)$$

$$= 2 \times 10^{-3} \text{ T}$$

The direction is given by the right-hand thumb rule.

b. The magnetic moment

$$m = NIA = NI\pi r^2 = 100 \times 3.2 \times 3.14 \times 10^{-2} = 10 \text{ A m}^2$$

The direction is once again given by the right-hand thumb rule.

c. $\tau = |\mathbf{m} \times \mathbf{B}|$

$$= mB \sin \theta$$

Initially, $\theta = 0$. Thus, initial torque $\tau_i = 0$. Finally, $\theta = \pi/2$ (or 90°).

Thus, final torque $\tau_f = mB = 10 \times 2 = 20 \text{ N m}$.

d. From Newton's second law, moment of inertia $I = 0.1 \text{ kg m}^2$

$$I \frac{d\omega}{dt} = mB \sin \theta$$

where g is the moment of inertia of the coil. From chain rule,

$$\frac{d\omega}{dt} = \frac{d\omega}{d\theta} \frac{d\theta}{dt} = \frac{d\omega}{d\theta} \omega$$

Using this,

$$I\omega d\omega = mB \sin \theta d\theta$$

Integrating from $\theta = 0$ to $\theta = \frac{\pi}{2}$,

$$I \int_0^{\omega_f} \omega d\omega = mB \int_0^{\pi/2} \sin \theta d\theta$$

$$I \frac{\omega_f^2}{2} = -mB \cos \theta \Big|_0^{\pi/2} = mB \text{ hence, angular speed acquired by the coil is given by :-}$$

$$\omega_f = \left(\frac{2mB}{I} \right)^{1/2} = \left(\frac{2 \times 20}{0.1} \right)^{1/2} = 20 \text{ rads}^{-1}$$

OR

a. As per Biot-Savart law, the magnetic field due to a current element $d\vec{l}$ at the observation point whose position vector \vec{r} is given by

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{l} \times \vec{r}}{r^3}$$

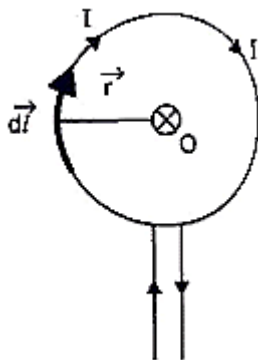
Where μ_0 is the permeability of free space.

Consider a circular loop of wire of radius r carrying a current I and also a current element dl of the loop. The direction of dl is along the tangent, so $dl \perp r$. From Biot-Savart law, magnetic field at the centre O due to the current element is

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin 90^\circ}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dl}{r^2}$$

The magnetic field due to all such current elements will point into the plane of paper at the centre O .

Hence the total magnetic field at the centre O is given by



$$B = \int dB = \int \frac{\mu_0 I dl}{4\pi r^2}$$

$$\text{or } B = \frac{\mu_0 I}{4\pi r^2} \int dl = \frac{\mu_0 I}{4\pi r^2} \cdot l$$

$$= \frac{\mu_0 I}{4\pi r^2} \cdot 2\pi r \text{ or } B = \frac{\mu_0 I}{2r}$$

$$\text{For a coil of } N \text{ turns, } B = \frac{\mu_0 NI}{2r}$$

b. Magnetic field at O due to loop 1,

$$B_1 = \frac{\mu_0 i R^2}{2(x^2 + R^2)^{3/2}} \text{ acting towards left}$$

Magnetic field at O due to loop 2,

$$B_2 = \frac{\mu_0 i R^2}{2(x^2 + R^2)^{3/2}} \text{ acting vertically upwards}$$

Here R is the radius of each loop.

Resultant field at O will be

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{2}B_1 (\because B_1 = B_2)$$

$$= \frac{\mu_0}{\sqrt{2}} \frac{iR^2}{(x^2 + R^2)^{3/2}}$$

This field acts at an angle of 45° with the axis of loop 1.

38. Number of horizontal wires in the telephone cable, $n = 4$

Current in each wire, $I = 1.0 \text{ A}$

Earth's magnetic field at a location, $H = 0.39 \text{ G} = 0.39 \times 10^{-4} \text{ T}$

The angle of dip at the location, $\delta = 35^\circ$

The angle of declination, $\theta \sim 0^\circ$

For a point 4cm below the cable:

Distance, $r = 4\text{cm} = 0.04\text{m}$

The horizontal component of the earth's magnetic field can be written as:

$$H_h = H \cos \delta - B$$

Where,

$B =$ Magnetic field at 4cm due to current I in the four wires

$$= 4 \times \frac{\mu_0 I}{2\pi r}$$

$\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7} \text{ Tm A}^{-1}$

$$\therefore B = 4 \times \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.04}$$

$$= 0.2 \times 10^{-4} \text{ T} = 0.2 \text{ G}$$

$$\therefore H_h = 0.39 \cos 35^\circ - 0.2$$

$$= 0.39 \times 0.819 - 0.2 \approx 0.12 \text{ G}$$

The vertical component of earth's magnetic field is given as:

$$H_v = H \sin \delta$$

$$= 0.39 \sin 35^\circ = 0.22 \text{ G}$$

The angle made by the field with its horizontal component is given as:

$$\theta = \tan^{-1} \frac{H_v}{H_h}$$

$$= \tan^{-1} \frac{0.22}{0.12} = 61.39^\circ$$

The resultant field at the point is given as:

$$H = \sqrt{(H_v)^2 + (H_h)^2}$$

$$= \sqrt{(0.22)^2 + (0.12)^2} = 0.25 \text{ G}$$

OR

i. The torque on the needle is $\vec{\tau} = \vec{M} \times \vec{B}$.

In magnitude, $\tau = MB \sin \theta$

Here, τ is restoring torque and θ is the angle between M and B .

Therefore, in equilibrium, restoring torque = deflecting torque i.e.

$$I \frac{d^2\theta}{dt^2} = -MB \sin \theta$$

Negative sign with $MB \sin \theta$ implies that the restoring torque is in opposite in direction to the deflecting torque.

For small values of θ in radians, we approximate $\sin \theta = \theta$ and we get

$$I \frac{d^2\theta}{dt^2} = -MB\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{MB}{I}\theta$$

$$\Rightarrow d^2\theta/dt^2 = -\omega^2\theta$$

$$\Rightarrow \alpha = -\omega^2\theta$$

where α being angular acceleration.

This is the general equation of simple harmonic motion with angular frequency ω .

The square of the angular frequency is

$$\omega^2 = \frac{MB}{I}$$

$$\omega = \sqrt{MB/I}$$

Time period for this simple harmonic motion, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB}}$

ii. a. As, the horizontal component of earth's magnetic field,

$$B_H = B \cos \delta, \text{ where } \delta \text{ being the angle of dip}$$

Putting $\delta = 90^\circ$ (as compass needle orients itself vertically), we get,

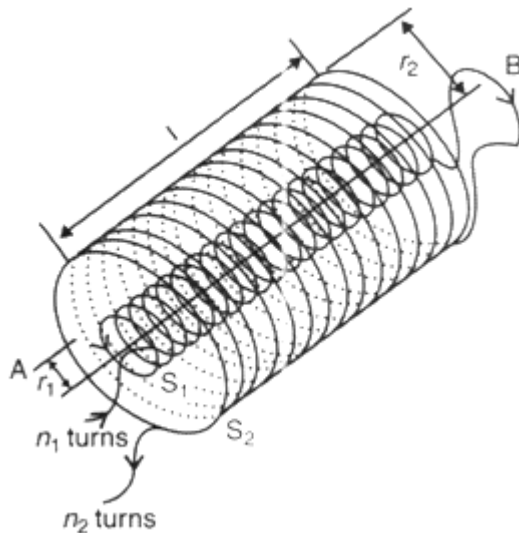
$$B_H = 0$$

b. For a compass needle oriented itself with its axis vertical at a certain place, the angle of dip, $\delta = 90^\circ$.

39. i. When two coils (or solenoids) are mutually coupled, then change in current of one coil causes a change in magnetic flux in the other coil and as a result an emf is induced in the other coil. This process is called mutual induction. Or **Mutual Inductance** is the interaction of one coils magnetic field on another coil as it induces a voltage in the adjacent coil.

SI unit of mutual inductance = Henry (H)

ii.



Let a current I_2 flow in the secondary coil

$$\therefore B_2 = \frac{\mu_0 N_2 i_2}{l}$$

$$\therefore \text{Flux linked with the primary coil} = \frac{\mu_0 N_2 N_1 A_1 i_2}{l}$$

$$= M_{12} i_2$$

$$\text{Hence, } M_{12} = \frac{\mu_0 N_2 N_1 A_1}{l}$$

$$\mu_0 n_2 n_1 A_1 l \left(n_1 = \frac{N_1}{l}; n_2 = \frac{N_2}{l} \right)$$

iii. Let a magnetic flux be (ϕ_1) linked with coil C_1 due to current (I_2) in coil C_2 .

We have:

$$\phi_1 \propto I_2$$

$$\Rightarrow \phi_1 = M I_2$$

$$\therefore \frac{d\phi_1}{dt} = M \frac{dI_2}{dt}$$

$$\Rightarrow e = -M \frac{dI_2}{dt}$$

OR

i. **Faraday's Laws of Electromagnetic Induction:** Faraday's First Law of Electromagnetic Induction states that whenever a conductor is placed in varying magnetic field emf is induced which is known as induced emf. If the conductor circuit is closed current is also induced which is called induced current.

Faraday's Second Law of Electromagnetic Induction states that the induced emf is equal to the rate of change of flux linkage where flux linkage is nothing but the product of the number of turns in the coil and flux associated with the coil.

$$\varepsilon = \frac{-d\phi_B}{dt}$$

ε_B is the magnetic flux through the circuit as $\phi_B = \int \vec{B} \cdot d\vec{A}$

With N loops of similar area in a circuit and ϕ_B being the flux through a loop, then emf is induced in every loop making Faraday law as

$$\varepsilon = -N \frac{\Delta\phi}{\Delta t}$$

where, ε = Induced emf [V],

N = number of turns in the coil

$\Delta\phi$ = change in the magnetic flux [Wb],

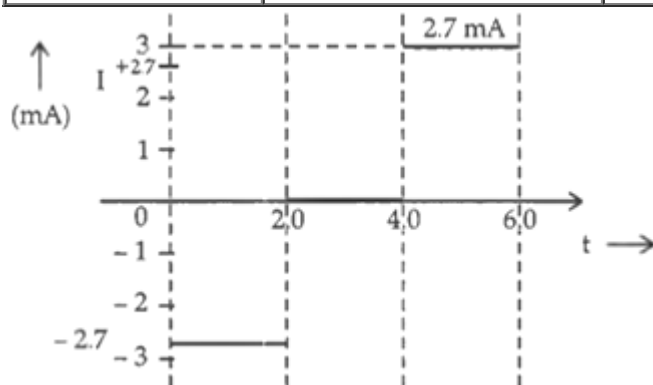
Δt = change in time [s]

The negative sign means that e opposes its cause.

$$\begin{aligned} \text{ii. } \varepsilon &= -\frac{d\phi}{dt} \\ &= -\pi R^2 \times \frac{dB}{dt} \\ &= -\frac{22}{7} \times (0.12)^2 \times \frac{1}{2} \\ \varepsilon &= -0.023 \text{ V,} \\ I &= \frac{\varepsilon}{R} \\ &= -2.7 \text{ mA for } 0 < t < 2\text{s.} \end{aligned}$$

Similarly,

	$0 < t < 2\text{s}$	$2 < t < 4\text{s}$	$4 < t < 6\text{s}$
ε (V)	-0.023	0	+0.023
I (mA)	-2.7	0	+2.7



iii. Lenz law states that the induced current always tends to oppose the cause which produces it. This extra work leads to periodic change in magnetic flux hence more current is induced. Thus the extra effort is just transformed into electrical energy which is law of conservation of energy. or

The energy of the field increase with the magnitude of the field. Lenz's law infers that there is an opposite field created due to increase of magnetic flux around a conductor so as to hold the law of conservation of energy.