Solution

Class 12 - Mathematics

2020-21 paper 4

Part A

1. we have, $\int_0^c 3dx = \frac{16}{3}$

$3(x)_0^c = \frac{16}{3}$

$3c = \frac{16}{3}$

$c = \frac{16}{9}$

2. according to given question , $lim_{x \to \infty}\left(\frac{1^2}{1^3+x^3} + \frac{2^2}{2^3+x^3} + ... + \frac{1}{2x}\right)$

$= lim_{x \to \infty}\left(\frac{1^2}{1^3+x^3} + \frac{2^2}{2^3+x^3} + ... + \frac{x^2}{x^3+x^3}\right)$

$= lim_{x \to \infty}\sum_{r=1}^{x}\left(\frac{r^2}{r^3+x^3}\right)$

$= \lim_{x \to \infty}\frac{1}{x}\sum_{r=1}^{x}\frac{\left(\frac{r}{x}\right)^2}{\left((\frac{r}{x})^3+1\right)}$

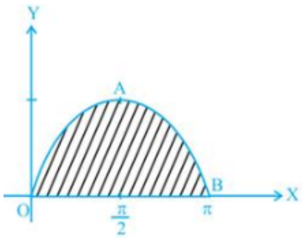
$= \int_0^1 \frac{x^2}{x^3+1}dx$

$= \frac{1}{3}[log\ (x^3 + 1)]_0^1$

$= \frac{1}{3}(log(1^3 + 1) - log(0^3 + 1)$

$= \frac{1}{3}(log\ 2 - log\ 1)[\ \because log1 = 0]$

$= \frac{1}{3}\ log\ 2$

3. 

We have

Area $OAB = \int_0^{\pi} ydx = \int_0^{\pi} \sin x dx = |-\cos x|_0^{\pi}$

$= \cos 0 - \cos\pi = 2$sq units.

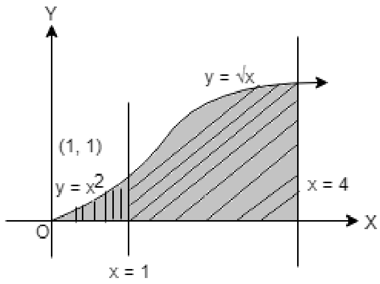
4. Required area = the area above x-axis ,bounded by the line x = 4 and the curve y = f(x) ,where f(x) $= x^2$, $0 \le x \le 1$ and f(x)=$\sqrt{x}, x \ge 1$

$=\int_0^1 x^2\ dx + \int_1^4 \sqrt{x}\ dx$

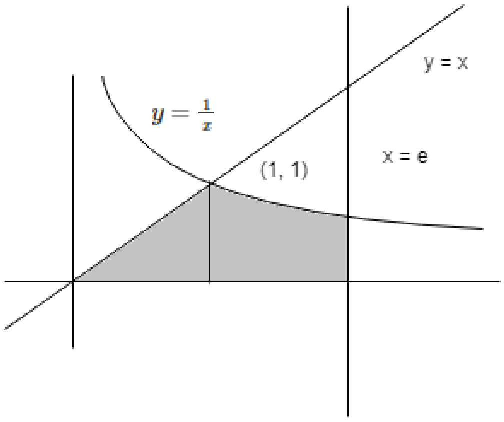
$=\left(\frac{x^3}{3}\right)_0^1 + \left(\frac{2x^{3/2}}{3}\right)_1^4$

$=\frac{1}{3} + \frac{2}{3}(8 - 1)$

= 5 sq units



5. We have $y = 4x^2$ *and* $y = \frac{1}{9}x^2$



Required area $=2\int_0^2\left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy$

$= 2\left(\frac{5y}{2}\frac{\sqrt{y}}{3/2}\right)_0^2$

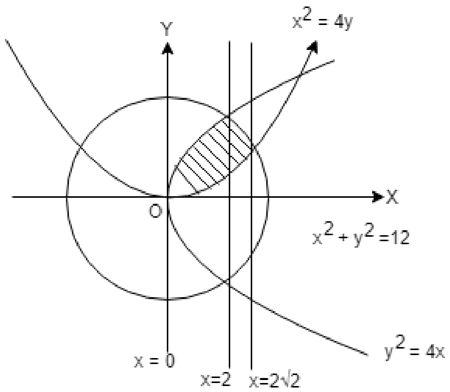
$= 2.\frac{5}{3}2\sqrt{2} = \frac{20\sqrt{2}}{3}$

6. Required area = the area lying in the first quadrant inside the circle $x^2 + y^2 = 12$ and bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

$=\int_0^2 2\sqrt{x}dx + \int_2^{2\sqrt{2}}\sqrt{12 - x^2}dx - \int_0^{2\sqrt{2}}\frac{x^2}{4}dx$

$=\left(\frac{4x^{3/2}}{3}\right)_0^2 + \left(\frac{x}{2}\sqrt{12 - x^2} + \frac{12}{2}sin^{-1}\frac{x}{2\sqrt{3}}\right)_2^{2\sqrt{2}} - \frac{1}{4}\left(\frac{x^3}{3}\right)_0^{2\sqrt{2}}$

$= 4\left(\frac{\sqrt{2}}{3} + \frac{3}{2}sin^{-1}\frac{1}{3}\right)$sq units



7. Let I $= \int_0^1\frac{2x}{5x^2+1}dx$ Then,we have,

I $= \frac{1}{5}\int_0^1\frac{10x}{5x^2+1}dx = \frac{1}{5}\left[\log\left(5x^2 + 1\right)\right]_0^1$

$= \frac{1}{5}(\log6 - \log1) = \frac{1}{5}\log6$

8. $I = \int \frac{x^3}{x+2} dx$

$= \int (x^2 - 2x + 4 - \frac{8}{x+2}) dx$ [Using long division method, we obtain]

$= \int \left\{ x^2 - 2x + 4 - \frac{8}{x+2} \right\} dx$

$= \frac{x^3}{3} - 2\frac{x^2}{2} + 4x - 8 \log |x + 2| + C$

$= \frac{x^3}{3} - x^2 + 4x - 8 \log |x+2| + c$

9. $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{(-2+1)}}{(-2+1)} + C = -\frac{1}{x} + C$, where C is constant of integration.

10. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int x^{\frac{5}{3}} dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$

$= \frac{3x^{\frac{8}{3}}}{8} + c$, where c is constant of integration.

11. $I = \int e^{-x} \text{cosec}^2 (2 e^{-x} + 5) dx$

Put $2e^{-x} + 5 = t$

$\Rightarrow -2e^{-x} dx = dt$

$\Rightarrow e^{-x} dx = -\frac{dt}{2}$

$\therefore I = \frac{1}{2} \int \text{cosec}^2 t \, dt$

$= \frac{1}{2} \cot t + c$

$= \frac{1}{2} \cot (2e^{-x} + 5) + C$

12. $I = \int a^x e^x dx$

$= \int (ae)^x dx$

$= \frac{(ae)^x}{\log ae} + c$

$I = \frac{(ae)^x}{\log ae} + c$

13. $\int \frac{1}{(\sqrt{x} + x)} dx = \int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx$

Now, let $(1 + \sqrt{x}) = t$ so that $\frac{1}{\sqrt{x}} dx = 2dt$

$\therefore \int \frac{1}{(\sqrt{x} + x)} dx = \int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx$

$= 2\int \frac{1}{t} dt = 2\log|t| + C = 2\log|(1 + \sqrt{x})| + C$ $[(1 + \sqrt{x}) = t]$

14. Let $I = \int \frac{x^2 - 1}{x^4 + 1} dx$

Re-writing the given integral as

$I = \int \frac{1 - \frac{1}{x^2}}{x^2 - \frac{1}{x^2}} dx$

$= \int \frac{1 - \frac{1}{x^2}}{\left(x + \frac{1}{x}\right)^2 - 2} dx$

Assume $t = x + \frac{1}{x}$

$dt = (1 - \frac{1}{x^2}) dx$

$\therefore I = \int \frac{dt}{t^2 - 2}$

Using identity $\int \frac{dz}{(z)^2 - 1} = \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| + c$

$\therefore\ I = \frac{1}{2\sqrt{2}}\log\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{y}+\sqrt{2}} + c$

15. I = $\int \tan^{\frac{3}{2}} x \sec^2 x dx$

Put tan x = t

$\Rightarrow \sec^2 x\ dx = dt$

$I = \int t^{\frac{3}{2}} dt$

$= \frac{2}{5} t^{\frac{5}{2}}$ +c

$= \frac{2}{5}(\tan x)^{\frac{5}{2}}$ +c

$\therefore\ I = \frac{2}{5}(\tan x)^{\frac{5}{2}}$+c

16. Take 5 + 6x = t

So we get

6 dx = dt

It can be written as

$\int \cos t\left(\frac{dt}{6}\right) = \frac{1}{6}\int \cos t dt$

By integrating w.r.t. t

$= \frac{1}{6} \times (\sin t) + c$

By substituting the value of t

$= \frac{\sin\ (5+6x)}{6} + c$

Section I

17. **(c)** - x cot x - $\frac{x^2}{2}$ + log |sin x| + C

Explanation: I = ∫x (cosec^2x - 1) dx = $\int \underset{I}{x}\ \underset{II}{cosec^2 x} dx - \int$ x dx

= x (-cot x) - ∫ (-cot x) dx - $\frac{x^2}{2}$ + C = -x cot x + log |sin x| - $\frac{x^2}{2}$ + C

18. **(d)** $\sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + C$

Explanation: (2 + 2x - x^2) = 3 - (1 + x^2 - 2x) $= (\sqrt{3})^2 - (x-1)^2$

$\therefore\quad I = \int \frac{dx}{\sqrt{(\sqrt{3})^2 - (x-1)^2}} = \int \frac{dt}{\sqrt{(\sqrt{3})^2 - t^2}}$, where (x - 1) = t and dx = dt

$= \sin^{-1}\frac{t}{\sqrt{3}} + C = \sin^{-1}\frac{(x-1)}{\sqrt{3}} + C$

19. **(c)** $\sin^{-1}\sqrt{x} - \sqrt{x(1-x)} + c$

Explanation: $I = \int\sqrt{\frac{x}{1-x}}dx$

$I = \int\sqrt{\frac{x}{1-x} \times \frac{x}{x}}dx$

$I = \int \frac{xdx}{\sqrt{x-x^2}}$

consider,

$x = A\frac{d\left(x-x^2\right)}{dx} + B$

$x = A(1-2x) + B$

$x = -2Ax + A + B$

$-2A = 1 \Rightarrow A = \frac{-1}{2}$

$\Rightarrow A + B = 0 \Rightarrow B = \frac{1}{2}$

$I = \int \frac{\frac{-1}{2}(1-2x) + \frac{1}{2}}{\sqrt{x-x^2}} dx$

$I = \int \left(\frac{-1}{2} \frac{1-2x}{\sqrt{x-x^2}} + \frac{1}{2\sqrt{x-x^2}} \right) dx$

$I = \frac{-1}{2} \times 2\sqrt{x-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{x-x^2}} dx$

Second term after completing square method you will get as

$I = -\sqrt{x-x^2} + \sin^{-1}\sqrt{x} + c$

20. **(d)** $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + C$

Explanation: On dividing Nr and Dr by x^2, we get

$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}\right)} dx = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left\{\left(x-\frac{1}{x}\right)^2+2\right\}} dx = \frac{dt}{t^2+2}$, where $x - \frac{1}{x} = t$

$= \int \frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} + C = \frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(x^2-1\right)}{\sqrt{2}x} + C$

21. **(a)** 1

Explanation: $\int_0^2 y dx = \frac{3}{\log_e 2}$

$\int_0^2 2^{kx} dx = \frac{3}{\log_e 2}$

$\left[\frac{2^{kx}}{\log_e 2}\right]_0^2 = \frac{3}{\log_e 2}$

$2^{2k} - 1 = 3$

$2^{2k} = 4$

$2^{2k} = 2^2$

$\Rightarrow 2k = 2$

$\Rightarrow k = 1$

22. **(d)** 2

Explanation: The graph of modulus function is V-shaped graph. Therefore , from graph , the area is = $\sqrt{2} \times \sqrt{2} = 2$.

23. **(b)** πab

Explanation: Area of standard ellipse is given by :πab.

24. **(b)** 9

Explanation: To find area the curves y = $\sqrt{x}$ and x = 2y + 3 and x – axis in the first quadrant., We have ;
$y^2 - 2y - 3 = 0$,(y – 3) (y + 1) = 0 . y = 3 , - 1. In first quadrant , y = 3 and x = 9.
Therefore , required area is ;

$\int_0^9 \sqrt{x} dx - \int_3^9 \left(\frac{x-3}{2}\right) = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9 = 9$

Section II

25. Let I = $\int \frac{x-3}{x^2+2x-4}dx$

Also let x - 3 = $\lambda\frac{d}{dx}(x^2 + 2x - 4) + \mu$

= $\lambda(2x + 2) + \mu$

x - 3 = $(2\lambda)x + (2\lambda + \mu)$

Comparing the coefficients of like powers of x, we get

$2\lambda = 2 \Rightarrow \lambda = \frac{1}{2}$

$\lambda + \mu = -3 \Rightarrow 6(\frac{1}{2}) + \mu = -3$

$\mu = -4$

So, $I = \int \frac{\frac{1}{2}(2x+2)-4}{x^2+2x-4}dx$

$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{x^2+2x+(1)^2-(1)^2-4}dx$

$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{(x+1)^2-(\sqrt{5})}dx$

I = $\frac{1}{2}\log|x + 2x - 4| - 4 \times \frac{1}{2\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + C$

[Since, $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log|\frac{x-a}{x+a}|$ + c]

I = $\frac{1}{2}\log|x + 2x - 4| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + c$

26. y = |x + 3|

$\Rightarrow y = (x+3), \text{ if } x \geq -3$

y = -(x + 3), if x < -3

$\int_{-6}^{0}|x+3|dx = ?$

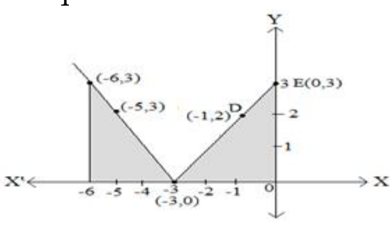
Area $= \int_{-6}^{-3} -(x+3)dx + \int_{-3}^{0}(x+3)dx$

$= \left[-\frac{x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$

$= \left[(-\frac{9}{2} + 9) - (-\frac{36}{2} + 18)\right] + \left[(0 + 0) - (\frac{9}{2} - 9)\right]$

$= \left[(\frac{9}{2} + 0) + (0 + \frac{9}{2})\right]$

= 9 sq units



OR

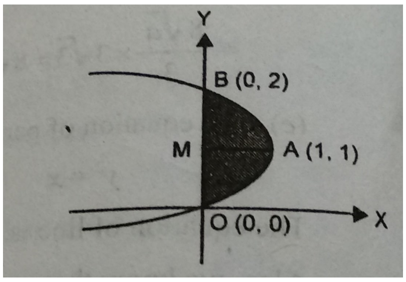
The given curve is $y^2 = 2y - x$ ....(1)

or $y^2 - 2y = -x$

or $y^2 - 2y + 1 = -x + 1$

or $(y-1)^2 = -(x-1)$,

which is a left handed parabola with vertex at (1,1).



putting x=0 in (1),we get,

$y^2 - 2y = 0$

$y(y - 2) = 0$

y = 0 ,2

Therefore,curve meets Y-axis in O(0,0) , B(0,2)

Required area=2(area OAM)

$=2\int_0^1 (2y - y^2)dy$

$=2\left[y^2 - \frac{y^3}{3}\right]_0^1$

$=2\left[(1 - \frac{1}{3}) - (0 - 0)\right]$

$=\frac{4}{3}$ sq.units.

27. Here both the functions viz. x and sin 3 x are easily integrable and the derivative of x is one, a less complicated function. Therefore, we take x as the first function and sin 3x as the second function.

Let I = $\int \underset{I}{x} \underset{II}{\sin 3x}\, dx$, then we have

I = x{∫ sin 3x dx} - ∫{$\frac{d}{dx}(x)$ × ∫ sin 3x dx} dx

⇒ I = $x \times -\frac{1}{3}$ cos 3x -∫ {$-\frac{1}{3}$ cos 3x} dx

⇒ I = $-\frac{1}{3}$ x cos 3x + $\frac{1}{3}$ ∫cos 3x dx = $-\frac{1}{3}$ x cos 3x + $\frac{1}{9}$ sin 3x + C

OR

We have

$\int(a \tan x + b \cot x)^2 dx = \int(a^2 \tan^2 x + b^2 \cot^2 x + 2ab \tan x \cot x)dx$

$= \int[a^2(\sec^2 x - 1) + b^2(\text{cosec}^2 x - 1) + 2ab]\, dx$

$= \int[a^2(\sec^2 x - 1) + b^2(\text{cosec}^2 x - 1) + 2ab]\, dx$

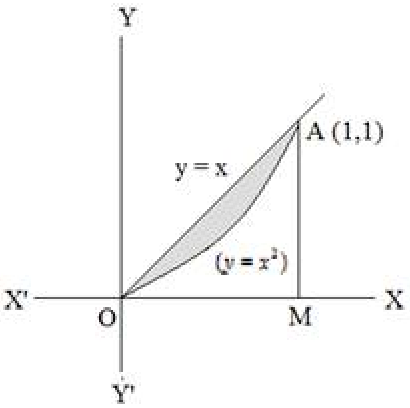
$= \int[a^2\sec^2 x - a^2 + b^2 \text{cosec}^2 x - b^2 + 2ab]\, dx$

$= a^2 \tan x - a^2 x - b^2 \cot x - b^2 x + 2ab\, x + C$

$= a^2 \tan x - b^2 \cot x - (a^2 - 2ab + b^2)x + C$

$= a^2 \tan x + -b^2 \cot x - (a-b)^2 x + C$

$\therefore \int(a \tan x + b \cot x)^2 dx = a^2 \tan x - b^2 \cot x - (a-b)^2 x + c$

28. 

$y = x^2$

y = x

$\Rightarrow x = 0, y = 0$

x = 1, y = 1

Area $= \int_0^1 x dx - \int_0^1 x^2 dx.$

$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$ sq. units

OR

Let I = $\int \frac{\log x}{(1+\log x)^2}$ dx

Also let log x = t

Then, x = e^t ⇒ dx = d(e^t) = e^t dt

$\therefore \quad I = \int \frac{te^t}{(t+1)^2}dt = \int \frac{(t+1)-1}{(t+1)^2}e^t dt$

$\Rightarrow I = \int\left\{\underset{f}{\frac{1}{t+1}} + \underset{f'}{\frac{-1}{(t+1)^2}}\right\}e^t dt$

$\Rightarrow I = \int \underset{I}{\frac{1}{t+1}}\underset{II}{e^t}dt + \int \frac{-1}{(t+1)^2}e^t dt$

$\Rightarrow I = \frac{1}{t+1}e^t - \int \frac{-1}{(t+1)^2}e^t dt + \int \frac{-1}{(t+1)^2}$ e^t dt + C

$\Rightarrow I = \frac{e^t}{t+1} + C = \frac{x}{(\log x+1)}$ + C

29. Let I = $\int \frac{1}{\sqrt{7-3x-2x^2}}dx$, then

$I = \int \frac{1}{\sqrt{-2\left[x^2+\frac{3}{2}x-\frac{7}{2}\right]}}dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[x^2+2x\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-\frac{7}{2}\right]}}dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x-\frac{3}{4}\right)^2-\frac{65}{16}\right]}}dx$

$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-\left(x+\frac{3}{4}\right)^2}}dx$

Let $\left(x + \frac{3}{4}\right) = t$

Then dx = dt

$I = \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-t^2}}dt$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{4}}\right) + c$ [Since $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$]

$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4\left(x + \frac{3}{4}\right)}{\sqrt{65}}\right) + c$

$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4x + 3}{\sqrt{65}}\right) + c$

30. Let I = $\int_0^{x/2} \frac{dx}{(1 - 2\sin x)}$, then

I = $\int_0^{\pi/2} \frac{dx}{1 - 2\left\{\frac{2\tan(x/2)}{1 + \tan^2(x/2)}\right\}}$

$= \int_0^{\pi/2} \frac{\sec^2(x/2)}{\left[1 + \tan^2(x/2) - 4\tan(x/2)\right]} dx$

$= 2\int_0^1 \frac{dt}{\left(1 + t^2 - 4t\right)}$, where tan $\frac{x}{2}$ = t $\left[x = 0 \Rightarrow t = 0 \; and \; x = \frac{\pi}{2} \Rightarrow t = 1\right]$

$= 2\int_0^1 \frac{dt}{(t - 2)^2 - (\sqrt{3})^2} = 2 \cdot \frac{1}{2\sqrt{3}}\left[\log\left|\frac{t - 2 - \sqrt{3}}{t - 2 + \sqrt{3}}\right|\right]_0^1$

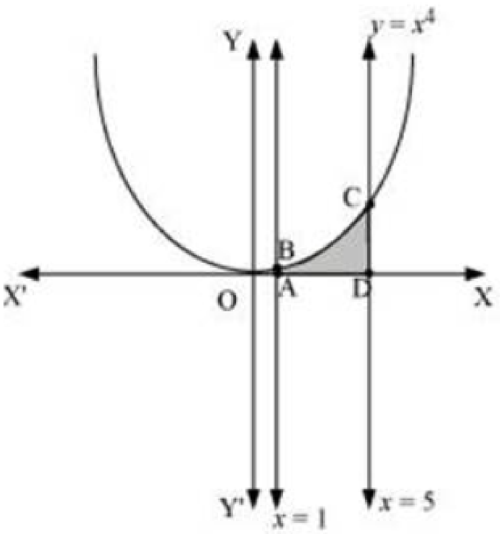
$= \frac{1}{\sqrt{3}}\left[\log\left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1}\right) - \log\frac{\sqrt{3} + 2}{\sqrt{3} - 2}\right]$

31. Let I = $\int \frac{3 - 5\sin x}{\cos^2 x} dx$, then

I = $\int \frac{3 - 5\sin x}{\cos^2 x} dx = 3\int \sec^2 x dx - 5\int \sec x \tan x \, dx$

= 3 tan x - 5 sec x + C

32.



We can see from the figure that the area of the region bounded by the curve y = x^4 and the lines x = 1, x = 5 is shown by shaded region that is Area ADCBA.

Area of ADCBA = $\int_1^5 y dy = \int_1^5 x^4 dx$

$\Rightarrow \left[\frac{x^5}{5}\right]_1^5$

$\Rightarrow\ 625 - \frac{1}{5}$

= 624.8 sq. units.

Which is the required solution.

33. $4x^2 + 4y^2 = 9$ ...(1)

$y^2 = 4x$ ...(2)

On Solving (1) and (2)

$y = \frac{1}{2}$

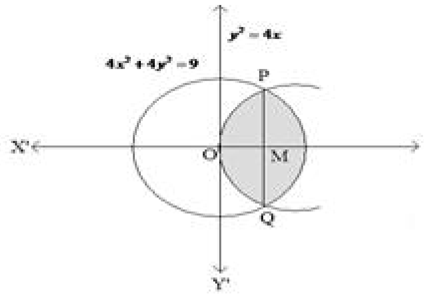
$= 2\left(\int_0^{1/2} 2\sqrt{y}dx + \int_{1/2}^{3/2}\sqrt{\frac{9}{4} - y^2}dx\right)$

$= 2\left[\frac{4}{3}(y^{\frac{3}{2}})_0^{1/2} + \left(\frac{y}{2}\sqrt{9/4 - y^2}\right)_{1/2}^{3/2}\right]$

$= 2\left[\frac{4}{3}\frac{1}{2\sqrt{2}} + \left(\frac{9\pi}{16} - \frac{1}{4}\sqrt{2} - \frac{9}{8}sin^{-1}(\frac{1}{3})\right)\right]$

$= \frac{8}{6\sqrt{2}} + \frac{9\pi}{8} - \frac{\sqrt{2}}{2} - \frac{9}{4}sin^{-1}(\frac{1}{3})$

$= \left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right]$sq units

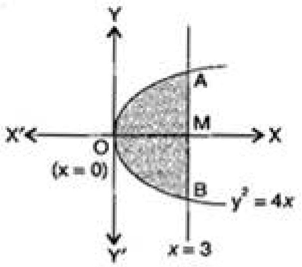


34. Equation of the (parabola) curve is

$y^2 = 4x$ ...(i)

$\Rightarrow y = 2\sqrt{x}$ ...(ii)

Here required shaded area OAMB = 2 × Area OAM



$= 2\left|\int_0^3 ydx\right| = 2\left|\int_0^3 2x^{\frac{1}{2}}dx\right| = 4\left|\frac{\left(x^{\frac{3}{2}}\right)_0^3}{\frac{3}{2}}\right|$

$= 4.\frac{2}{3}\left[3^{\frac{3}{2}} - 0\right] = \frac{8}{3}.3\sqrt{3} = 8\sqrt{3}$ sq. units

Section III

35. We have to show $\int_0^a f(a-x)dx = \int_0^a f(x)dx$

Take $I = \int_0^a f(a-x)dx$ ......(i)

Let a - x = z $\Rightarrow$ -dx = dz $\Rightarrow$ dx - dz
Also, x = 0 $\Rightarrow$ z = a and x = a $\Rightarrow$ z = 0

$\therefore$ I = $\int_a^0 f(z)(-dz) = \int_a^0 f(z)dz$

$\Rightarrow$ I = $\int_0^a f(z)dz$ ($\because$ $\int_a^b f(x)dx = -\int_0^a f(x)dx$

$\Rightarrow$ I = $\int_0^a f(x)dx$ .....(ii) ($\because$ $\int_a^b f(z)dz = \int_a^b f(x)dx$

From (i) and (ii), we get

$\int_0^a (a-x)dx = \int_0^a f(x)dx$

Now, $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x}$ dx ......(iii)

$$\Rightarrow \text{I} = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{x}{2}-x\right)+\cos\left(\frac{x}{2}-x\right)}dx = \frac{\pi}{2\sqrt{2}}\int \frac{1}{\cos x\frac{1}{\sqrt{2}}+\sin x\frac{1}{\sqrt{2}}}dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{\sin x \sin\frac{\pi}{4} + \cos x \cos\frac{\pi}{4}}dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)}dx$$

$$2\text{I} = \frac{\pi}{2\sqrt{2}}\int_0^{\frac{\pi}{2}} \sec\left(x + \frac{\pi}{4}\right)dx$$

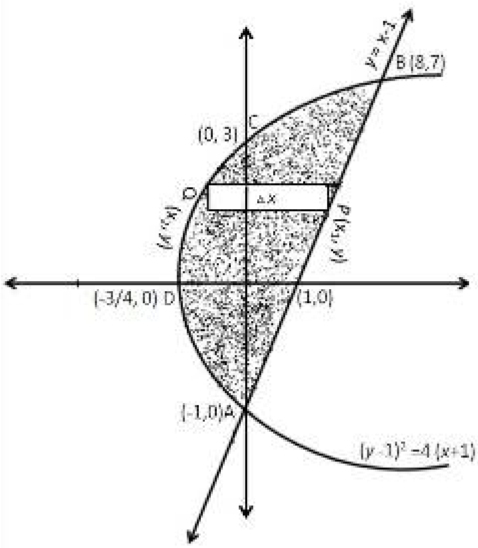
$$= \frac{\pi}{2\sqrt{2}}\left[\log(\sec x + \tan x)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2\sqrt{2}}\left[\log\left(\sec\frac{\pi}{2} + \tan\frac{\pi}{2}\right) - [\log(\sec 0 + \tan 0)]\right]$$

$$2\text{I} = \frac{\pi}{2\sqrt{2}}[\infty - \log 1] = \infty$$

I = ∞

36. To find region bounded by curves
y = x - 1 ...(i)
and $(y - 1)^2 = 4(x + 1)$ ...(ii)
Equation (i) represents a line passing through (1, 0) and (0, -1) equation (ii) represens a parabola with vertex (-1, 1) passes through (0, 3), (0, -1), $\left(-\frac{3}{4}, 0\right)$
Their points of intersection (0, -1) and (8, 7)
A rough sketch of curves is given as:-



Shaded region is required area. It is sliced in rectangles of area $(x_1 - x_2)\triangle y$.
It slides from y = -1 to y = 7, so
Required area of the shaded region= Area of the Region ABCDA

$A = \int_{-1}^{7}\left(x_1 - x_2\right)dy$

$= \int_{-1}^{7}\left(y + 1 - \frac{(y-1)^2}{4} + 1\right)dy$

$= \frac{1}{4}\int_{-1}^{7}$ (4y + 4 - y^2 - 1 + 2y + 4)dy

$= \frac{1}{4}\int_{-1}^{7}$ (6y + 7 - y^2) dy

$= \frac{1}{4}\left[3y^2 + 7y - \frac{y^3}{3}\right]_{-1}^{7}$

$= \frac{1}{4}\left[\left(147 + 49 - \frac{343}{3}\right) - \left(3 - 7 + \frac{1}{3}\right)\right]$

$= \frac{1}{4}\left[\frac{245}{3} + \frac{11}{3}\right]$

$A = \frac{64}{3}$ sq units

37. Let the given integral be,

$I = \int\sin^{-1}\left\{\frac{2x+2}{\sqrt{4x^2+8x+13}}\right\}dx$

$= \sin^{-1}\left\{\frac{2x+2}{\sqrt{(2x+2)^2+3^2}}\right\}dx$

Substituting 2x + 2 = 3 tan θ and dx = $\frac{3}{2}\sec^2\theta\, d\theta$, we get

$I = \int\sin^{-1}\left(\frac{3\tan\theta}{3\sec\theta}\right) \times \frac{3}{2}\sec^2\theta d\theta = \frac{3}{2}\int\underset{I}{\theta}\ \underset{II}{\sec^2\theta}d\theta$

$\Rightarrow\quad I = \frac{3}{2}\{\theta\tan\theta - \int\tan\theta d\theta\} = \frac{3}{2}\{\theta\tan\theta - \log|\sec\theta|\}$

$\Rightarrow\quad I = \frac{3}{2}\left\{\left(\frac{2x+2}{3}\right)\tan^{-1}\left(\frac{2x+2}{3}\right) - \log\sqrt{1 + \left(\frac{2x+2}{3}\right)^2}\right\} + C$

$\Rightarrow\quad I = \frac{3}{2}\left\{\left(\frac{2x+2}{3}\right)\tan^{-1}\left(\frac{2x+2}{3}\right) - \log\sqrt{4x^2 + 8x + 13}\right\} + C$

$\Rightarrow$ I = (x + 1) $\tan^{-1}\left(\frac{2x+2}{3}\right) - \frac{3}{4}$ log ($4x^2$ + 8x +13) + C

OR

Let $I = \int\sin^{-1}\sqrt{\frac{x}{a+x}}dx$

Put $x = a\tan^2\theta$

$\Rightarrow dx = 2a\tan\theta\sec^2\theta d\theta$

$\therefore I = \int\sin^{-1}\sqrt{\frac{a\tan^2\theta}{a + a\tan^2\theta}}\left(2a\tan\theta.\sec^2\theta\right)d\theta$

$= 2a\int\sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)\tan\theta.\sec^2\theta d\theta$

$= 2a\int\sin^{-1}(\sin\theta)\tan\theta.\sec^2\theta d\theta$

$= 2a\int\theta.\tan\theta\sec^2\theta d\theta$

$= 2a\left[\theta.\int\tan\theta.\sec^2\theta d\theta - \int\left(\frac{d}{d\theta}\theta.\int\tan\theta.\sec^2\theta d\theta\right)d\theta\right]$

Let $\tan\theta = t$

$\sec^2\theta d\theta = dt$

$\int \tan\theta \sec^2\theta d\theta = \int t dt = \frac{t^2}{2} = \frac{\tan^2\theta}{2}$

$I = 2a\left[\theta . \frac{\tan^2\theta}{2} - \int \frac{\tan^2\theta}{2} d\theta\right]$

$= a\theta\tan^2\theta - a\int\left(\sec^2\theta - 1\right)d\theta$

$= a\theta\tan^2\theta - a\tan\theta + a\theta + C$

$= a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1}\sqrt{\frac{x}{a}}\right] + C$

38. Given, $\lim_{x\to\infty} \frac{(1^2+2^2+3^2+......+x^2)(1^3+2^3+3^3+.....+x^3)}{(1^6+2^6+3^6+....+x^6)}$

$= \lim_{x\to\infty} \frac{\sum_{r=1}^{x} r^2 . \sum_{r=1}^{x} r^3}{\sum_{r=1}^{x} r^6}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}\sum_{r=1}^{x}(\frac{r}{x})^2 . \frac{1}{x}\sum_{r=1}^{x}(\frac{r}{x})^3}{\frac{1}{x}\sum_{r=1}^{x}(\frac{r}{x})^6}$

$= \frac{\int_0^1 y^2\, dy . \int_0^1 y^3\, dy}{\int_0^1 y^6 dy}$

$= \frac{\frac{1}{3} \cdot \frac{1}{4}}{\frac{1}{7}}$

$= \frac{7}{12}$

39. $I = \int\left(\frac{2+2\sin x . \cos x}{2\cos^2 x}\right)e^x dx$

$\int\left(\frac{2}{2\cos^2 x} + \frac{2\sin x . \cos x}{2\cos^2 x}\right)e^x dx$

$= \int(\sec^2 x + \tan x)e^x dx$

Let f(x) = tanx

$f'(x) = \sec^2 x$

$\because$ We know that $\int e^x\left[f(x) + f'(x)\right]dx = e^x f(x) + c$

$\therefore \int(\sec^2 x + \tan x)e^x dx$

$= e^x . \tan x + c$

40. Let the given integral be,

$I = \int(3x + 1)\sqrt{4 - 3x - 2x^2}dx$

Let $(3x + 1) = A\frac{d}{dx}\left(4 - 3x - 2x^2\right) + B$

$\Rightarrow$ (3x + 1) = A(-3 - 4x) + B

$\Rightarrow$ (3x + 1) = -4Ax + (B - 3A)

$\Rightarrow$ 3 = -4A and (B - 3A) = 1

$\Rightarrow$ A = $-\frac{3}{4}$ and B = $-\frac{5}{4}$

$\Rightarrow$ $I = -\frac{3}{4}\int(-3 - 4x)\sqrt{4 - 3x - 2x^2}dx - \frac{5}{4}\int\sqrt{4 - 3x - 2x^2}dx$

Let $I = -\frac{3}{4}I_1 - \frac{5}{4}I_2$ .....(i)

Now,

$I_1 = \int(-3 - 4x)\sqrt{4 - 3x - 2x^2}dx$

Let $(4 - 3x - 2x^2) = t$ or $(-3 - 4x)dx = dt$

$\Rightarrow\ I_1 = \int \sqrt{t}dt$

$= \frac{2}{3}t^{\frac{3}{2}} + c_1$

$\Rightarrow\ I_1 = \frac{2}{3}\left(4 - 3x - 2x^2\right)^{\frac{3}{2}} + c_1$

Now, $I_2 = \int \sqrt{4 - 3x - 2x^2}dx$

$= \int \sqrt{2\left(2 - \frac{3}{2}x - x^2\right)}dx$

$= \sqrt{2}\int \sqrt{\left(\frac{17}{4} - \frac{9}{4} - \frac{3}{2}x - x^2\right)}dx$

$= \sqrt{2}\int \sqrt{\left[\left(\frac{\sqrt{17}}{2}\right)^2 - \left(\frac{9}{4} + \frac{3}{2}x + x^2\right)\right]}dx$

$= \sqrt{2}\int \sqrt{\left[\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x + \frac{3}{2}\right)^2\right]}dx$

$= \sqrt{2}\sin\left(\frac{x + \frac{3}{2}}{\frac{\sqrt{17}}{2}}\right) + c_2$

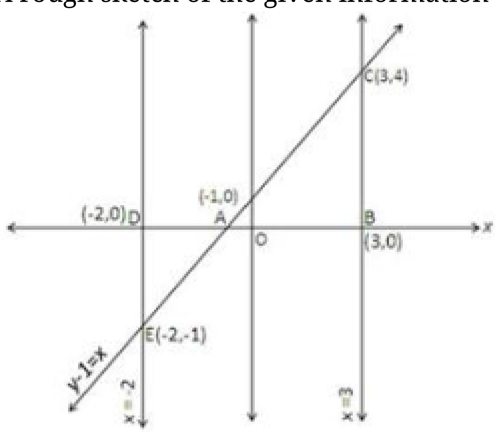
$= \sqrt{2}\sin\left(\frac{2x + 3}{\sqrt{17}}\right) + c_2$

Using (i), we get

$I = -\frac{3}{4} \times \frac{2}{3}\left(4 - 3x - 2x^2\right)^{\frac{3}{2}} - \frac{5}{4} \times \sqrt{2}\sin\left(\frac{2x + 3}{\sqrt{17}}\right) + C$

$\therefore\ I = -\frac{1}{2}\left(4 - 3x - 2x^2\right)^{\frac{3}{2}} - \frac{5\sqrt{2}}{4}\sin\left(\frac{2x + 3}{\sqrt{17}}\right) + C$

41. To find area of region bounded by x-axis the ordinates x = -2 and x = 3 and y -1 = x ...(i)
Equation (i) is a line that meets at axes at (0, 1) and (-1, 0)
A rough sketch of the given information is as under:-



Bounded region provides the required area.
Now Required area = Area of Region ABCA + Area of Region ADEA

$A = \int_{-1}^{3} ydx + \left|\int_{-2}^{-1} ydx\right|$

$= \int_{-1}^{3}(x + 1)dx + \left|\left[- 2^{-1}(x + 1)dx \right|\right.$

$= \left(\frac{x^2}{2} + x\right)_{-1}^{3} + \left|\frac{x^2}{2} + x\right|_{-2}^{-1}$

$= \left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right] + \left(\frac{1}{2} - 1\right) - (2 - 2)|$

$= \left[\frac{15}{2} + \frac{1}{2}\right] + \left|-\frac{1}{2}\right|$

$= 8 + \frac{1}{2}$

$A = \frac{17}{2}$ sq. units

OR

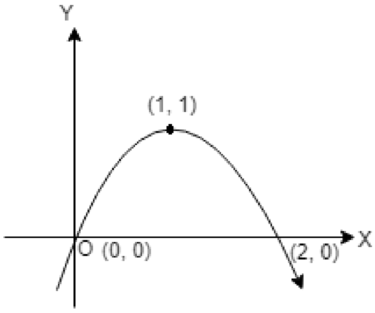
The equation of curve is

$y = 2x - x^2$............(1)

$\Rightarrow x^2 - 2x = -y$

$\Rightarrow x^2 - 2x + 1 = -y + 1$

$(x - 1)^2 = -(y - 1)$,which is a downward parabola with vertex (1,1).



putting y = 0 in (1),we get, $0 = 2x - x^2$,

$\therefore\ x(x - 2) = 0$

$x = 0, 2$

$\therefore$ parabola meets x-axis at (0,0),(2,0)

$\therefore$ required area= Area bounded by the curve $y = 2x - x^2$ and the x-axis

$=\int_0^1 y\,dx$

$=\int_0^2 (2x - x^2)dx$

$=\left[x^2 - \frac{x^3}{3}\right]_0^2$

$=\left[4 - \frac{8}{3} - (0 - 0)\right)]$

$= \frac{4}{3} sq\ units$

Section IV

42. $\frac{2x}{x^2+3x+2}$

$= \frac{2x}{x^2+2x+x+2}$

$= \frac{2x}{x(x+2)+1(x+2)}$

$= \frac{2x}{(x+1)(x+2)}$

$= \frac{A}{x+1} + \frac{B}{x+2}$ ....(i)

$\Rightarrow 2x = A(x + 2) + B(x + 1)$

$\Rightarrow 2x = Ax + 2A + Bx + B$

Comparing coefficients of x on both sides A + B = 2 ....(ii)

Comparing constants 2A + B = 0 .....(iii)

Solving eq. (ii) and (iii), we get A = -2 and B = 4

Putting these values of A and B in eq. (i),

$$\frac{2x}{(x+1)(x+2)} = \frac{-2}{x+1} + \frac{4}{x+2}$$

$$\therefore \int\frac{2x}{(x+1)(x+2)}dx = -2\int\frac{1}{x+1}dx + 4\int\frac{1}{x+2}dx$$

= -2 log |x + 1| + 4 log |x + 2| + c

= 4 log |x + 2| - 2 log |x + 1| + c

OR

The given integral is

$$I = \int\frac{1}{\sin^4 x\cos^2 x}dx \text{ ...(i)}$$

Then, I = $\int \sin^{-4} x \cos^{-2} x dx$

Since -4 - 2 = -6, which is even integer. So, we divide both numerator and denominator by $\cos^6$ x.

$$\therefore\ I = \int\frac{\frac{1}{\cos^6 x}}{\frac{\sin^4 x\cos^2 x}{\cos^6 x}}dx$$

$$= \int\frac{\sec^6 x}{\frac{\sin^4 x}{\cos^4 x}}dx$$

$$= \int\frac{\sec^6 x}{\tan^4 x}dx$$

$$= \int\frac{\sec^4 x\times\sec^2 x}{\tan^4 x}dx$$

$$= \int\frac{\left(\sec^2 x\right)^2\times\sec^2 x}{\tan^4 x}dx$$

$$= \int\frac{\left(1+\tan^2 x\right)^2\times\sec^2 x}{\tan^4 x}dx$$

$$\Rightarrow\quad I = \int\frac{\left(1+\tan^4 x+2\tan^2 x\right)\times\sec^2 x}{\tan^4 x}dx \text{ ...(ii)}$$

by using substitution.

Let tan x = t. Then,

d (tan x) = dt

$\Rightarrow$ $\sec^2$ xdx = dt

$$\Rightarrow\quad dx = \frac{dt}{\sec^2 x}$$

Putting tan x = t and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int\frac{\left(1+t^4+2t^2\right)}{t^4}\times\sec^2 x\times\frac{dt}{\sec^2 x}$$

$\int (t^{-4} + 1 + 2t^{-2})$ dt

$$= -\frac{t^{-3}}{3} + t - 2t^{-1} + c$$

$$= -\frac{1}{3t^3} + t - \frac{2}{t} + c$$

$$= -\frac{1}{3\tan^3 x} + \tan x - \frac{2}{\tan x} + c$$

$$= -\frac{1}{3}\times cot^3 + \tan x - 2x\cot x + c$$

$$\therefore\quad I = \frac{-1}{3}\times\cot^3 x - 2\cot x + \tan x + c$$

43. According to the question ,

Given parabola is y^2 = x......(i)

vertex of parabola is (0, 0)

axis of parabola lies along X-axis.

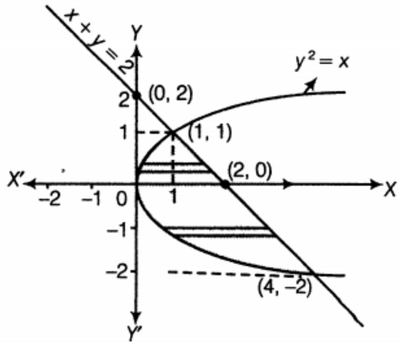
Given equation of line is x + y = 2.......(ii)

For, x + y = 2

x	2	0
y	0	2

So, line passes through the points (2, 0) and (0, 2).

Now, let us sketch the graph of given curve and line as shown below:



On putting x = 2 - y from Eq. (ii) in Eq. (i), we get

$y^2 = 2 - y$

$\Rightarrow y^2 + y - 2 = 0$

$\Rightarrow y^2 + 2y - y - 2 = 0$

$\Rightarrow y(y + 2) - 1(y + 2) = 0$

$\Rightarrow (y - 1)(y + 2) = 0$

$\therefore$ y = 1 or -2

When y = 1, then x = 2 - y = 1

When y = -2, then x = 2 - y = 2 - (- 2) = 4

So, points of intersection are (1, 1) and (4, - 2).

Now, required area = $\int_{-2}^{1} \left[x_{(line)} - x_{\text{(parabola)}}\right] dy$

$= \int_{-2}^{1} \left(2 - y - y^2\right) dy$

$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$

$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)$

$= 2 - \frac{5}{6} + 6 - \frac{8}{3}$

$= 8 - \frac{5}{6} - \frac{8}{3}$

$= \frac{48 - 5 - 16}{6}$

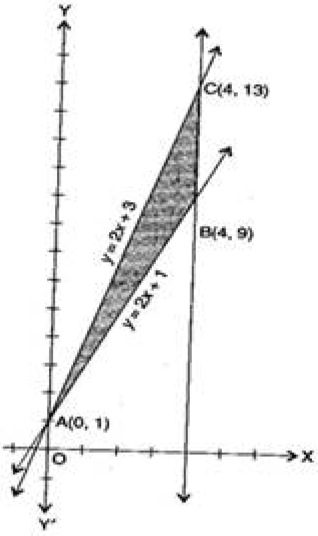
$= \frac{48 - 21}{6}$

$= \frac{27}{6}$

$= \frac{9}{2}$ sq units.

OR

Equations of one side of triangle is



y = 2x + 1 ...(i)
second line of triangle is y = 3x + 1 ...(ii)
third line of triangle is x = 4 ...(iii)
Solving eq. (i) and (ii), we get x = 0 and y = 1
∴ Point of intersection of lines (i) and (ii) is A (0, 1)
Putting x = 4 in eq. (i), we get y = 9
∴ Point of intersection of lines (i) and (iii) is B (4, 9)
Putting x = 4 in eq. (i), we get y = 13
∴ Point of intersection of lines (ii) and (iii) is C (4, 13)
∴ Area between line (ii) i.e., AC and x - axis

$$= \left|\int_0^4 y\,dx\right| = \left|\int_0^4 (3x+1)dx\right| = \left(\frac{3x^2}{2} + x\right)_0^4$$

= 24 + 4 = 28 sq. units ...(iv)
Again Area between line (i) i.e., AB and x - axis

$$= \left|\int_0^4 y\,dx\right| = \left|\int_0^4 (2x+1)dx\right| = \left(x^2 + x\right)_0^4$$

= 16 + 4 = 20 sq. units ...(v)
Therefore, Required area of $\triangle ABC$
= Area given by (iv) – Area given by (v)
= 28 – 20 = 8 sq. units

44. Let I = $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\sin x + \cos x)dx$ ....(i)

Using the property: $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$\Rightarrow \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\sin x + \cos x)dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\sin(-x) + \cos(-x))dx$

I = $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\cos x - \sin x)dx$ ....(ii)

Adding equation (i) and (ii)

2I = $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\sin x + \cos x)dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\cos x - \sin x)dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\cos^2 x - \sin^2 x\right)dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\cos 2x)dx$

As cos(-x) = cos x

Using property: $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ (for f(-x) = f(x))

$\Rightarrow 2I = 2\int_0^{\frac{\pi}{4}} \log\cos 2x dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\cos 2x dx$ .... (iii)

Now, by property $\int_0^a f(a-x)dx = \int_0^a f(x)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left(\cos 2\left[\frac{\pi}{4} - x\right]\right)dx = \int_0^{\frac{\pi}{4}} \log\left(\cos\frac{\pi}{2} - 2x\right)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\sin 2x dx$ .... (iv)

Adding Equations, (iii) and (iv), we get

$\Rightarrow 2I = \int_0^{\frac{\pi}{4}} (\log\sin 2x + \log\cos 2x dx)$

$= \int_0^{\frac{\pi}{4}} \log(\sin 2x\cos 2x)dx$

$= \int_0^{\frac{\pi}{4}} \log\left(\frac{1}{2}\sin 4x\right)dx$

$= \int_0^{\frac{\pi}{4}} \log\sin 4x dx + \int_0^{\frac{\pi}{4}} \log\frac{1}{2} dx$

$= \int_0^{\frac{\pi}{4}} \log\sin 4x dx + [x]_0^{\frac{\pi}{4}} \log\frac{1}{2}$

$= \int_0^{\frac{\pi}{4}} \log\sin 4x dx + \frac{\pi}{4}\log\frac{1}{2}$

$\Rightarrow 2I = I_1 + \frac{\pi}{4}\log\frac{1}{2}$

Where, $I_1 = \int_0^{\frac{\pi}{4}} \log\sin 4x dx$

Let 2x = t

$\Rightarrow 2dx = dt$

When x = 0, t = 0 and at $x = \frac{\pi}{4}, t = \frac{\pi}{2}$

$\Rightarrow I_1 = \frac{1}{2}\int_0^{\frac{\pi}{2}} \log\sin 2t dt$

$\Rightarrow I_1 = \frac{1}{2}2\int_0^{\frac{\pi}{4}} \log\sin 2t dt$ [Using $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$]

$= \int_0^{\frac{\pi}{4}} \log\sin 2t dt$

$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \log\sin 2x dx$ ...(by change of variable property)

Hence, $2I = \int_0^{\frac{\pi}{4}} \log\sin 2x dx + \frac{\pi}{4}\log\frac{1}{2}$

$\Rightarrow 2I = I + \frac{\pi}{4}\log\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{4}\log\frac{1}{2}$

OR

Let the given integral be, $y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$ .... (i)

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)}{1+\sin\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)}dx$$

$$y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1+\sin x}dx \text{ ..... (ii)}$$

Adding eq. (i) and eq.(ii)

$$2y = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1+\sin x}dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x}dx$$

Using rationalization, we have

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{\cos^2 x}dx$$

$$y = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} [\sec^2 x - \frac{\sin x}{\cos^2 x}]dx$$

Let, cos x = t

$\Rightarrow$ -sin x dx = dt

At $x = \frac{\pi}{4}, t = \frac{1}{\sqrt{2}}$

At $x = \frac{3\pi}{4}, t = \frac{1}{\sqrt{2}}$

$$y = \frac{\pi}{2}\left([\tan x]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + \int_{\sqrt{2}}^{\frac{-1}{\sqrt{2}}} \frac{1}{t^2}dt \right)$$

$$y = \frac{\pi}{2}\left(\tan\frac{3\pi}{4} - \tan\frac{\pi}{4} + \left(\frac{-1}{t}\right)_{\frac{1}{\sqrt{2}}}^{\frac{-1}{\sqrt{2}}} \right.$$

$$y = \frac{\pi}{2}(-1 - 1 + \sqrt{2} + \sqrt{2}) = \pi(\sqrt{2} - 1)$$

Hence proved.