Solution

Class 10 - Mathematics

2020-2021 - Paper-4

Part - A

1. Given that area of the circle = circumference
   $\pi r^2 = 2\pi r$
   $\Rightarrow \pi r^2 - 2\pi r = 0$
   $\Rightarrow \pi r(r-2) = 0$
   $\Rightarrow \mathbf{r} = 0$ or $\mathbf{r} = 2$
   Since radius cannot be equal to 0 so, r = 2
2. Area of circle = $\pi \times 50^2 = 2500\pi$
   New radius = 50 - $\frac{50}{100} \times 50$= 25 cm
   Area of new circle = 625π cm^2
   Decrease in area = 2500π - 625π
   = 1875π cm^2
   Percentage decrease in area = $\frac{1875\pi}{2500\pi} \times 100$= 75%
3. Perimeter of a semi-circular protactor = Perimeter of a semi-circle = $\frac{1}{2}$(circumference of circle) + diameter = $\frac{1}{2}$(circumference of circle) + 2 $\times$ radius = $(2r + \pi r)cm$
   $\Rightarrow 2r + \pi r = 36$ [Given, perimeter of semi-cicular protactor = 36]
   $\Rightarrow r = \frac{36}{2+\pi}$
   $\Rightarrow r = 7cm$
   Hence, diameter of semi-circular protactor = 2r = 2(7) = 14cm
4. We know that the area A of a sector of a circle of radius r and central angle θ(in degrees) is given by
   $A = \frac{\theta}{360} \times \pi r^2$
   Here, r = 28 cm and θ = 45.
   $\therefore A = \frac{45}{360} \times \pi \times (28)^2 = \frac{1}{8} \times \frac{22}{7} \times 28 \times 28\text{cm}^2 = 308\text{cm}^2$
5. Lateral surface area of a cylinder = 94.2 cm^2
   h = 5 cm
   $2\pi rh = 94.2$
   $\Rightarrow 2 \times 3.14 \times r \times 5$= 94.2
   $\Rightarrow r = \frac{94.2}{2\times 3.14\times 5}$ = 3 cm
6. Length of resulting cuboid, l = 4 cm + 4 cm = 8 cm, breadth b = 4 cm, & height h = 4 cm
   We know that,
   Surface area of cuboid
   = 2 (lb + bh + hl)
   = 2 (8×4 + 4×4 + 4×8)
   = 2 × 80 = 160 cm^2.
7. Let the edges of the cube be x_1 and x_2 respectively.
   $\frac{x_1^3}{x_2^3} = \frac{1}{64} \Rightarrow \frac{x_1}{x_2} = \frac{1}{4}$
   $\Rightarrow 4x_1 = x_2$
   Ratio of their surface areas
   $= \frac{6x_1^2}{6x_2^2} = \frac{x_1^2}{x_2^2} = \frac{x_1^2}{(4x_1)^2}$
   $= \frac{x_1^2}{16x_1^2}$= 1 : 16
8. Surface area of a sphere = 616 cm^2.
   $\Rightarrow \quad 4\pi r^2 = 616$

$r^2 = \frac{616}{4\pi} = \frac{616 \times 7}{4 \times 22}$
$\Rightarrow \quad r^2 = 7 \times 7$
$\Rightarrow \quad r = \sqrt{7 \times 7}$= 7 cm

9.

Less than	50	55	60	65	70	75	80
Frequency	0	2	10	22	46	84	100

10. Number of observations = 25

Hence, median = Value of $\left(\frac{n+1}{2}\right)^{th}$ Observation

= Value of $\left(\frac{25+1}{2}\right)^{th}$ Observations

= Value of 13^{th} Observation

11.

Height	Frequency	c.f.
$140 - 145$	5	5
$145 - 150$	15	$5 + 15 = 20$
$150 - 155$	25	$25 + 20 = 45$
$155 - 160$	30	$45 + 30 = 75$
$160 - 165$	15	$75 + 15 = 90$
$165 - 170$	10	$90 + 10 = 100$
	$\sum$f = 100	

$N = 100$
$\Rightarrow \frac{N}{2}$th term = $\frac{100}{2}$ = 50th term
Hence, Median class is 155 - 160.

12. Mode = 3 median - 2 mean
Mode = 12.4 and mean = 10.5
Median = $\frac{1}{3}$Mode + $\frac{2}{3}$Mean
= $\frac{1}{3}$(12.4) + $\frac{2}{3}$(10.5)
$= \frac{12.4}{3} + \frac{21}{3}$
$= \frac{12.4+21}{3}$
$= \frac{33.4}{3}$
$= 11.13$
So, median is 11.13.

13. In a deck of cards, there are 52 card
$\therefore$n = 52
One card is drawn at random
Prabibilty of the event = $\frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$
The Required probability = $\frac{12}{52} = \frac{3}{13}$

14. Total number events 1 to 6 = 6
Let K be the event of getting number greater than 2.
The numbers possible greater that 2 are: 3, 4, 5, 6
The number of outcomes favorable to K = 4
$\therefore \text{P(K)} = \frac{\text{No.of outcomes favorable to K}}{\text{Total No. of outcomes}} = \frac{4}{6} = \frac{2}{3}$
Hence the probability of getting number greater than 2 = $\frac{2}{3}$

15. We know that
November has 30 days, which means 4 weeks and 2 days.
Now, 4 weeks will contain 4 Sunday.
The remaining 2 days may be :

i. Sunday and Monday

ii. Monday and Tuesday
iii. Tuesday and Wednesday
iv. Wednesday and Thursday
v. Thursday and Friday
vi. Friday and Saturday
vii. Saturday and Sunday

Total number of possible outcomes = 7
Now, favourable outcomes are: Sunday and Monday, Saturday and Sunday
Number of favourable outcomes = 2
Required probability (P) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of possible\ outcomes}$
P = $\frac{2}{7}$

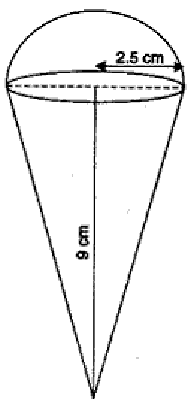
16. Total bulbs in a box = 400
Defective bulbs = 15
Non-defective bulbs = 400 -15 = 385
P (non-defective bulb) =$\frac{385}{400} = \frac{77}{80}$

Section I

17. i. (a) 0.2
ii. (b) 0.85
iii. (c) $\frac{3}{5}$
iv. (a) $\frac{x}{100+x}$
v. (c) $\frac{2}{3}$

18. i. (b) 240.625 cm^2
ii. (a) 160.42 cm^2
iii. (d) 110 cm
iv. (a) 9:1
v. (d) $\frac{2}{9}\pi r^2$

19. For cone, Radius of the base (r)
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$
Height (h) = 9 cm
$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$
$= \frac{825}{14}\text{cm}^3$



For hemisphere,
Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$
$\therefore$ Volume = $\frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$
i. (a) $\frac{1357}{42}\text{cm}^3$

ii. (d) The volume of the ice-cream without hemispherical end = Volume of the cone
$= \frac{825}{14}\text{cm}^3$

iii. (c) The TSA of the cone is given by:
$\pi rl + \pi r^2$

iv. (a) Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere
$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$
$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

v. (d) remain unaltered

20. i. (a) Curve A - Less than type ogive and Curve B - More than type ogive
ii. (c) Median Wages = 50 Rs.
iii. (d) Mode = 3 Median - 2 Mean = 3(50) - 2(50) = 50 Rs.
As, Mean = Median = Mode, so it is a symmetrical distribution
iv. (a) Median
v. (b) Median

Section II

21. Area of the shaded region= Area of quadrant DPBA + Area of quadrant DQBC - Area of a square ABCD
$= \left\{\left(\frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) + \left(\frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) - (7 \times 7)\right\}\text{cm}^2$
$= \left(\frac{77}{2} + \frac{77}{2} - 49\right)\text{cm}^2$
= (77 - 49) cm^2
= 28 cm^2

22. Perimeter of shaded region
= Perimeter of semicircle PSR + Perimeter of semicircle RTQ + Perimeter of semicircle PAQ
$\Rightarrow Perimeter = (5\pi + 1.5\pi + 3.5\pi)\text{cm}$
$= 10\pi\text{cm}$
$= 10 \times 3.14\text{cm}$
$= 31.4cm$

23. According to the question,we are given that,
Volume of brass = 2.2 dm^3 = 2200 cm^3
Diameter of cylindrical wire = 0.25 cm
Therefore, radius of wire = $\frac{0.25}{2}$ cm
By the given condition ,we have,
Volume of cylindrical wire = Volume of Brass
$\Rightarrow \quad \pi r^2 h = 2200$
$\Rightarrow \quad \frac{22}{7} \times \left(\frac{0.25}{2}\right)^2 \times h = 2200$
$\Rightarrow \quad h = \frac{2200 \times 7 \times 2 \times 2}{22 \times 0.25 \times 0.25}$
$\Rightarrow$ h = 44800 cm = 448 metres

OR

It is given that the mean of 100 observations is 50 . The value of the largest observation is 100.
Mean = $\frac{\Sigma fx}{\Sigma f}$
$\Rightarrow$ 50 = $\frac{\Sigma fx}{100}$
$\Rightarrow \quad \Sigma fx$ = 5000
It was later found that it is 110 not 100.
Correct, $\Sigma fx'$ = 5000 + 110 - 100 = 5010
Therefore, Correct Mean = $\frac{\Sigma fx'}{\Sigma f} = \frac{5010}{100}$ = 50.1
It is given that the median of 100 observations is 52.
Median will remain same i.e. = 52

24. According to the question,
Average age of parents 20 years ago = 23 years
Their total ages 20 years ago = 23 $\times$ 2 = 46 years
Let parents age = x years

Sum of present age of parents = 46 + 20 $\times$ 2 = 86 years

Average = $\frac{86+x}{3}$

$\Rightarrow \quad 34=\frac{86+x}{3}$

$\Rightarrow 102=86+x$

$\Rightarrow x=16$

Therefore my present age is 16 years.

25. Total number of cards = 52.
    i. let E_1 be the event of getting a red face card.
       4 kings, 4 queens and 4 jacks are all face cards.
       Number of red face cards = 2+ 2+ 2 = 6.
       number of favourable outcomes = 6.
       $\therefore$ P(getting a red face card) =$P(E_1)=\frac{6}{52}$=$\frac{3}{26}$
    ii. let E_2 be the event of getting a black king.
       Number of black kings = 2
       number of favourable outcomes = 2.
       $\therefore$ P(getting a black king) = $P(E_2)=\frac{2}{52}$=$\frac{1}{26}$.

OR

Total number of cards = 52

All jacks, queens, kings and aces of red colour are removed therefore, remaining cards = 52 - 2 -2 - 2 - 2=44

i. Favourable outcomes of a black queen = 2
   Probability of a black queen = $\frac{2}{44}$ = $\frac{1}{22}$
ii. Total red cards = 13 $\times$ 2 = 26
   Favourable outcomes of a red color = total red cards - red picture cards - red aces = 26 - 6 - 2 =18
   Probability of a red card = $\frac{18}{44}$=$\frac{9}{22}$
iii. Favourable outcomes of a black jack = 2
   Probability of a black jack = $\frac{2}{44}$=$\frac{1}{22}$
iv. Total number of picture cards in a suit = 4$\times$3 = 12
   Favourable outcomes of a picture card = total picture cards - red picture cards = 12 - 6 = 6
   Probability of a picture card = $\frac{6}{44}$=$\frac{3}{22}$

26. The Slant height of cone

$l=\sqrt{(120)^2+(160)^2}$

$=\sqrt{14400+25600}$

$=\sqrt{40000}$

= 200

Surface area of sphere = surface area of cone

$4\pi r_1^2$ = πrl

$\mathbf{r}_1^2=\frac{rl}{4}$

$r_1^2=\frac{120\times200}{4}$

r_1^2=6000

Radius of sphere

$r_1=\sqrt{6000}$

= 77.46

Part - B

27. After removing the king, queen and jack of clubs from a deck of 52 playing cards there are 49 cards left in the deck. Out of these 49 cards one card can be chosen in 49 ways.
    Total number of elementary events = 49
    i. There are 13 heart cards in the deck containing 49 cards out of which one heart card can be chosen in 13 ways.

Favourable number of elementary events = 13
Hence, P (Getting a heart) $= \frac{13}{49}$

ii. There are 3 kings in the deck containing 49 cards.
Out of these three kings one king can be chosen in 3 ways.
Favourable number of elementary events = 3
Hence, P (Getting a king)$= \frac{3}{49}$

iii. After removing king, queen and jack of clubs only 10 club cards are left in the deck.
out of these 10 club cards one club card is chosen in 10 ways.
Favourable number of elementary events = 10
Hence, P (Getting a club)$= \frac{10}{49}$

iv. There is only one '10' of hearts. Favourable number of elementary events = 1
Hence, P (Getting the '10' to hearts) $= \frac{1}{49}$

28. Calculation of mean:

Class interval	**Mid – value (x_i)**	**f_i**	**f_ix_i**
0 – 6	3	6	18
6 – 12	9	8	72
12 – 18	15	10	150
18 – 24	21	9	189
24 – 30	27	7	189
		Σf_i = 40	$\Sigma f_i x_i$ = 618

We know that, Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$
$= \frac{618}{40}$
$= 15.45$

OR

We have,

Class Intervals	**Frequency (f)**	**C.F**
Below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51
	$N = \sum f = 51$	

Here, $\frac{N}{2} = \frac{51}{2} = 25.5$ which is in the class 145-150
Here, l_1 = 145, h = 5, N = 51, C = 11, F = 18

$\therefore$ Median $= l_1 + \frac{\frac{N}{2} - C}{f} \times h$
$= 145 + \frac{25.5 - 11}{18} \times 5$
$= 145 + \frac{72.5}{18} \Rightarrow 149.03$
$\therefore$ Median height of the girls = 149.03

29. Suppose level of water rises by h metres in the pond. Then, clearly, water risen in the pond forms a cuboidal of dimensions $80\text{m} \times 80\text{m} \times \text{hm}$.
$\therefore \quad 80 \times 50 \times h$= Volume of water displaced by 500 persons
$\Rightarrow \quad 80 \times 50 \times h = 500 \times 0.04$

$\Rightarrow$ 4000h = 20

$\Rightarrow \quad h=\frac{1}{200}\mathrm{m}=0.5\mathrm{cm}$

30. Surface area of a solid metallic sphere = 616 cm^2.

$\Rightarrow \quad 4\pi r^2=616$

$\Rightarrow \quad 4\times\frac{22}{7}\times r^2=616$

$\Rightarrow \quad r^2=\frac{616\times 7}{4\times 22}$

$\Rightarrow \quad r^2=49$

$\Rightarrow \quad r=\sqrt{49}=7\mathrm{cm}$

Therefore, Radius of sphere = 7 cm

Height of cone = 28 cm

According to the question,

Volume of cone = Volume of sphere

$\Rightarrow \quad \frac{1}{3}\pi(r_1)^2\times 28=\frac{4}{3}\pi(7)^3$

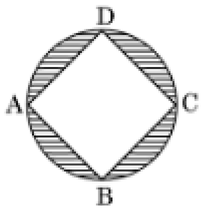
$\Rightarrow \quad r_1^2\times 28=4\times 7\times 7\times 7$

$\Rightarrow \quad r_1^2=49$

$\Rightarrow \quad r_1=\sqrt{49}=7\mathrm{cm}$

Therefore, Radius of cone = 7 cm

And Diameter of base of cone = 2(7) = 14 cm

31. 

Given that, ABCD is a square with side 22 cm.

To find radius join AC and DB. Let O be the intersecting point. Also O be the center of circle.

Hence OA, OB, OC, OD are radii of the circle. So, let OA = OB = r

By pythogoras theorem in triangle OAB, we get,

$AB^2=OA^2+OB^2$

$(22)^2=r^2+r^2$

$484=2r^2$

$\implies r^2=242$

$\implies r=\sqrt{2}42=11\sqrt{2}$

Hence, radius of circle is $11\sqrt{2}cm$

Thus, Shaded area = Area of circle - Area of square

= $\pi r^2-side\times side$

= $3.14\times(11\sqrt{2})^2-22\times 22$

= $3.14\times 242-484$

= 759.88 - 484

= 275.88 cm^2

32. According to the question,

Cost of fencing a circular field = Rs. 5500

Rate of fencing per metre = Rs. 25

$\therefore$ Perimeter of a circular field

$=\frac{\text{cost of fencing}}{\text{Rate per metre}}$

$=\left(\frac{5500}{25}\right)m$

= 220 m

Let r be the radius of the circular field.

Then, $2\pi r$ = 220

$\Rightarrow r=220\times\frac{7}{44}$

$\Rightarrow$ r = 35 m

Therefore, Area of the circular field $=\pi r^2$

$= \left(\frac{22}{7} \times 35 \times 35\right)$

= 3850 m^2

Cost of ploughing per m^2 = 50 paise

Therefore,Cost of ploughing 3850 m^2

= Rs. $\frac{50}{100} \times 3850$

= Rs.1925.

33. The total number of marbles = 54.

As per given condition

P(getting a blue marble) = $\frac{1}{3}$ and P(getting a green marble) = $\frac{4}{9}$

Let P(getting a white marble) be x.

Since, there are only 3 types of marbles in the jar, the sum of probabilities of all three marbles must be 1.

Therefore, $\frac{1}{3}+\frac{4}{9}$+x=1

$\Rightarrow \frac{3+4}{9}$+x=1

$\Rightarrow \frac{7}{9}$+x=1

$\Rightarrow$ x = $1-\frac{7}{9}$

$\Rightarrow$ x = $\frac{9-7}{9}$

$\Rightarrow$ x = $\frac{2}{9}$

Therefore, P(getting a white marble) = $\frac{2}{9}$ .... (1)

Let the number of white marbles be n.

Probability = $\frac{Number\ of\ favourable\ outcome}{Total\ Number\ of\ outcomes}$

Then, P(getting a white marbles) = $\frac{n}{54}$ .... (2)

From (1) and (2),

$\frac{n}{54} = \frac{2}{9}$

$\Rightarrow$ n = $\frac{2\times54}{9}$

$\Rightarrow$ n = $\frac{108}{9}$

$\Rightarrow$ n = 12

Thus, there are 12 white marbles in the jar.

OR

C.I.	**x_i**	**u_i**	**f_i**	**f_iu_i**
35-40	37.5	- 5	1	- 5
40-45	42.5	- 4	2	- 8
45-50	47.5	- 3	3	- 9
50-55	52.5	- 2	x	- 2x
55-60	57.5	- 1	y	- y
60-65	62.5 = A	0	6	0
65-70	67.5	1	8	8
70-75	72.5	2	4	8
75-80	77.5	3	2	6
80-85	82.5	4	3	12
85-90	87.5	5	2	10
Total			Σf_i = 31 + x + y	Σf_iu_i = 22 - 2x - y

Let Assumed Mean, A = 62.5

Here, Σf_i = 31 + x + y = 40

$\Rightarrow$ x + y = 9..........(i)

$\Sigma f_i u_i$ = 22 - 2x - y

Now, Mean = A + $\frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$\Rightarrow$ 63.5 = 62.5 + $\frac{(22-2x-y)}{40} \times 5$

$\Rightarrow$ 2x + y = 14...........(ii)

Solving eqns (i) and (ii), x = 5 and y = 4.

Section III

34. We have,

Diameter of the sphere = 6 cm

$\therefore$ Radius of the sphere = $\frac{6}{2}\text{cm} = 3\text{cm}$

$\Rightarrow$ Volume of the sphere == $\frac{4}{3} \times \pi \times 3^3\text{cm}^3 = 36\pi\text{cm}^3 \left[\text{ Using } V = \frac{4}{3}\pi r^3\right]$

Let the radius of cross-section of wire be r cm. It is given that the length of the cylindrical shaped wire is 36 m.

$\therefore$ Volume of the wire = $\left(\pi r^2 \times 3600\right)\text{cm}^3$

Since metallic sphere is converted into cylindrical shaped wire. Therefore, Volume of the wire = Volume of the sphere

$\Rightarrow \quad \pi r^2 \times 3600 = 36\pi$

$\Rightarrow \quad r^2 = \frac{36\pi}{3600\pi} = \frac{1}{100}$

$\Rightarrow \quad r = \frac{1}{10}\text{cm} = 1\text{mm}$

35. Total number of possible outcomes n = 100

i. $The\ even\ numbers\ from\ 1\ to\ 100 = \frac{100}{2} = 50$

$\therefore$ P (card taken out has an even number) = $= \frac{m}{n} = \frac{50}{100} = \frac{1}{2}$

ii. The multiples of 13 from 1 to 100 are: $13, 26, 39, 52, 65, 78, 91$

No. of favorable outcomes m = 7

$\therefore$P(card taken out has multiple of 13) = $= \frac{m}{n} = \frac{7}{50}$

iii. Perfect square number from 1 to 100 are :$1, 4, 9, 16, 25, 36, 49, 64, 81, 100$

No. of all favorable outcomes m = 10

$\therefore$ P(card taken out has a perfect square number) = $= \frac{m}{n} = \frac{10}{100} = \frac{1}{10}$

iv. Prime numbers less than 20 are : $2, 3, 5, 7, 11, 13, 17, 19$

No. of all favorable outcomes $m = 8$

$\therefore$ P(card taken out has a prime number less than 20) = $= \frac{m}{n} = \frac{8}{100} = \frac{2}{25}$

OR

p = Frequency of the class + cf of preceding class

$= 12 + 11 = 23$

q = cf of the class - cf of preceding class

$= 46 - 33 = 13$

Table:

Class Interval	Frequency	Cumulative Frequency
100 - 200	11	11
200 - 300	12	23
300 - 400	10	33
400 - 500	13	46
500 - 600	20	66
600 - 700	14	80

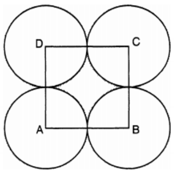
$N = 80 \Rightarrow \frac{N}{2} = 40$

The cumulative frequency just greater than 40 is 46.

Hence, median class is $400 - 500$.

Here, maximum frequency $= 20$

Hence, modal class is $500 - 600$.

36. 

Let r cm be the radius of each circle.

Area of square - Area of 4 sectors = $\frac{24}{7}\mathrm{cm}^2$

(side)^2 - $4\left[\frac{\theta}{360}\pi\mathrm{r}^2\right]$ = $\frac{24}{7}\mathrm{cm}^2$

or, $(2r)^2 - 4\left(\frac{90^\circ}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - 4\left(\frac{1}{4^\circ} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$

or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

or, $6r^2 = 24$

or, $r^2 = 4$

or, $r = \pm 2$

or, Radius of each circle is 2 cm (r cannot be negative)