Solution

Class 10 - Mathematics

2020-21 paper 5

Part-A

1. Our fraction terminates only when it's denominator is in form of 2^n5^m. So the expression is in the form of 2^n5^m

   Multiplying and dividing the expression by $5^3 \times 2^2$

   $\frac{23}{2^{352}} = \frac{23 \times 53 \times 2^2}{2^{352} \times 5^3 \times 2^2}$

   $= \frac{11500}{10^5} = 0.115$

   OR

   No.

   According to Euclid's division lemma,

   a = 3q + r, where $0 \leq r < 3$

   and r is an integer. Therefore, the values of r can be 0, 1 or 2.

2. Let the number of John's marbles be x.

   Therefore, number of Jivanti's marble = 45 − x

   After losing 5 marbles,

   Number of John's marbles = x − 5

   Number of Jivanti's marbles = 45 − x − 5 = 40 − x

   Given that the product of their marbles is 124.

   $\therefore$ (x – 5) (40 – x) = 124

   $\Rightarrow$ x^2 – 45x +324 = 0

3. No.

   The Condition for no solution is : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (parallel lines)

   Given pair of equations,

   x = 2y and y = 2x

   or x - 2y = 0 and 2x - y = 0;

   Comparing with ax + by + c = 0;

   Here, a_1 = 1, b_1 = - 2, c_1 = 0;

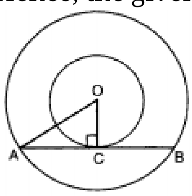
   And a_2 = 2, b_2 = - 1, c_2 = 0;

   a_1 / a_2 = 1/2

   b_1 / b_2 = -2/-1 = 2

   Here, $a_1/a_2 \neq b_1/b_2$.

   Hence, the given pair of linear equations has unique solution.

4. 

   Now, In $rt. \triangle OCA$

   $AO^2 = OC^2 + AC^2$

   $\Rightarrow AC^2 = 5^2 - 3^2$

   $\Rightarrow AC = 4$

   We know, $OC \perp AB$

   $\therefore AC = BC$

   Hence, $AC = 2(4) = 8cm$

5. Here, $a_n = 3n^2 + 5$

   Since nth term is not linear in n. So, it is not an A.P

OR

Let a be the first term and d be the common difference of the given AP. Then,

$S_n = \frac{n}{2} \cdot [2a+(n-1)d]$,

$\therefore 3(S_8 - S_4) = 3[\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)]$

$= 3[4(2a+7d) - 2(2a+3d)] = 6(2a+11d)$

$= \frac{12}{2} \cdot (2a+11d) = S_{12}$.

Hence, $S_{12} = 3(S_8 - S_4)$.

6. 0.6, 1.7, 2.8, 3.9...
   First term = a= 0.6
   Common difference (d) = Second term -First term
   = Third term - Second term and so on
   Therefore, Common difference (d) = 1.7−0.6=1.1
7. $x^2 - 2x = (-2)(3-x)$
   $\Rightarrow x^2 - 2x = -6 + 2x$
   $\Rightarrow x^2 - 2x - 2x + 6 = 0$
   $\Rightarrow x^2 - 4x + 6 = 0$
   Here, degree of equation is 2.
   Therefore, it is a Quadratic Equation.

OR

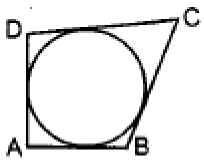
Given quadratic equation is $3x^2 - 4x + 1 = 0$.

$\therefore D = b^2 - 4ac = (-4)^2 - 4(3)(1) = 16 - 12 > 0$

Hence, the roots are real and distinct.

8. We know , tangent and radius of a circle are perpendicular to each other at point of contact
   $\therefore$ Angle between the radii $= 360^0 - 90^0 - 90^0 - 30^0 = 150^0$
9. PQ = PR = 5 cm (Length of Tangents from same external point are always equal)
   and PQ = QS (perpendicular from center of the circle to the chord bisects the chord)
   $\therefore$ PS = 2PQ
   $= 2 \times 5 = 10\text{cm}$

OR



Now,

$AB + CD = BC + AD$

$\Rightarrow 12 + 14 = 15 + \text{AD}$

$\Rightarrow AD = 11cm$

10. In △ CEB and △ CDA
    ∠C = ∠C [Common]
    ∠CEB = ∠CDA [Each 90°]
    ∴ △CEB ~ △CDA [AA − Similarity]
    then, $\frac{CE}{CD} = \frac{CB}{CA}$ [Corresponding sides of 2 similar triangles are proportional]
    $\Rightarrow CE \times CA = CB \times CD$
    Hence proved.
11. We have $a_1 = 11$, $a_2 = 22$ and $a_3 = 33$
    $a_2 - a_1 = 11$
    $a_3 - a_2 = 11$
    Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.
12. We know that $\sin^2\theta + \cos^2\theta = 1$
    Therefore, $(\sin^2\theta + \cos^2\theta)^3 = 1$

or, $(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) = 1$

or, $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

13. L.H.S. = $\tan^2\theta - \sin^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}\cdot\sin^2\theta$

= $\tan^2\theta\sin^2\theta$ = R.H.S.

Hence Proved

14. Thickness of the log = $\frac{\text{volume}}{l\times b} = \frac{1.104}{2.3\times0.75}$ m = 0.64 m

Number of planks = $\frac{\text{thickness of the log}}{\text{thickeness of one plank}} = \frac{0.64}{0.04}$ = 16

15. Since $(5x+2),(4x-1)$ *and* $(x+2)$ are in AP, we have

$(4x-1)-(5x+2)=(x+2)-(4x-1)$

$\Rightarrow 4x-1-5x-2=x+2-4x+1$

$\Rightarrow -x-3=-3x+3$

$\Rightarrow$ 2x=6

$\Rightarrow$ x=3.

16. In out of 52 playing cards, 4 jacks, 4 queens and 4 kings are removed, then the remaining cards are left,

n(S) = 52 - 3 $\times$ 4 = 40

Let E_1 = Event of getting a card whose value is 7

E = Card value 7 may be of a spade, a diamond, a club or a heart

So, $n(E_1)$ = 4

$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{40} = \frac{1}{10}$

OR

Two dice are thrown at the same time. [given]

So, total number of possible outcomes = 36

We have different number on both dice.

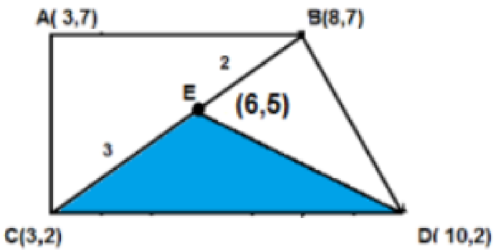
So, number of possible outcomes

= 36 { Number of possible outcomes for same number on both dice}

= 36 - 6 = 30

$\therefore$ Required probability = $\frac{30}{36} = \frac{5}{6}$

17. i. (b) Let us redraw the figure as follows:



Now, Coordinates of C and B are C(3, 2) and B(8, 7).

E divides CB internally in the ratio 3 : 2.

Therefore, the coordinates of E, by applying the section formula, are

$\left(\frac{3\times8+2\times3}{3+2}, \frac{3\times7+2\times2}{3+2}\right)$ = (6, 5)

ii. (c) Area of the $\triangle$ECD $= \frac{1}{2}[\,6(2-2)+3(2-5)+10(5-2)]$

$= \frac{1}{2}[3\times-3+10\times3]$ = 10.5 square unit

iii. (a) The coordinates of the flower plants of Ajay and Deepak are (3, 7) and (10, 2) respectively.

Therefore, the distance between plants of Ajay and Deepak $= \sqrt{(10-3)^2+(7-2)^2}$

$= \sqrt{74}$ = 8.60 unit

iv. (d) 5 units

v. (b) 7 units

18. i. (a) 26 m
ii. (c) Claim 1 would be correct if n>m, n>r and Claim 3 would be correct if ABJ is a right triangle.
iii. (c) PR^2 = PQ.RQ
iv. (d) $\frac{AB}{BD} = \frac{AJ}{AH}$
v. (c) 114 cm^2

19. i. (d) 60
ii. (b) 39.71
iii. (c) 11
iv. (b) frequency of the class preceding the modal class
v. (d) 3 Median = Mode + 2 Mean

20. i. (c) 8028 cm^3
ii. (b) 3900 cm^2
iii. (c) 214 cm
iv. (b) 24
v. (a) 8700 cm^2

Part-B

21. Let us assume that 6 + √2 is a rational number.
So we can write this number as
6 + √2 = a/b
Here a and b are two co-prime numbers and b is not equal to 0
Subtract 6 both side we get
√2 = a/b – 6
√2 = (a-6b)/b
Here a and b are integers so (a-6b)/b is a rational number. So √2 should be a rational number. But √2 is an irrational number. It is a contradiction.
Hence result is 6 + √2 is a irrational number

22. P(9a - 2, -b) divides the line segment joining A(3a + 1, -3) and B(8a, 5) in the ratio 3:1
Then, by section formula
Coordinates of $P = \left(\frac{3\times8a+1\times(3a+1)}{3+1}, \frac{3\times5+1\times(-3)}{3+1}\right)$
$\Rightarrow (9a-2,-b) = \left(\frac{24a+3a+1}{4}, \frac{15-3}{4}\right)$
$\Rightarrow (9a-2,-b) = \left(\frac{27a+1}{4}, 3\right)$
$\Rightarrow 9a-2 = \frac{27a+1}{4}$ and $-b = 3$
⇒ 36a - 8 = 27a + 1 and b = -3
⇒ 9a = 9
⇒ a = 1 and b = -3

OR

The points of trisection means that the points which divide the line into three equal parts. From the figure, it is clear that C, and D are these two points. Let C (x_1, y_1) and D (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., BC = CD = DA
Let BC = CD = DA = k, Point C divides BC and CA as: BC = kCA = CD + DA = k + k = 2k
Hence the ratio between BC and CA is: $\frac{BC}{CA} = \frac{k}{2k} = \frac{1}{2}$
Therefore, point C divides BA internally in the ratio 1:2 then by section formula we have that if a point P(x, y) divides two points P (x_1, y_1) and Q (x_2, y_2) in the ratio m:n then, the point (x, y) is given by
$(\mathrm{x},\mathrm{y}) = \left(\frac{\mathrm{mx_2+nx_1}}{\mathrm{m+n}}, \frac{\mathrm{my_2+ny_1}}{\mathrm{m+n}}\right)$
Therefore C(x, y) divides B(–2, –3) and A(4,–1) in the ratio 1:2, then
$C(x,y) = \left(\frac{(1\times4)+(2\times-2)}{1+2}, \frac{(1\times-1)+(2\times-3)}{1+2}\right)$

$C(x,y) = \left(\frac{4-4}{1+2}, \frac{-1-6}{1+2}\right)$

$C(x,y) = \left(0, \frac{-7}{3}\right)$

Point D divides the BD and DA as:DA = kBD = BC + CD = k + k = 2k

Hence the ratio between BD and DA is: $\frac{BD}{DA} = \frac{2k}{k} = \frac{2}{1}$

The point D divides the line BA in the ratio 2:1

So now applying section formula again we get,

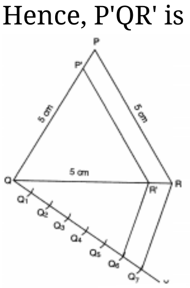
$D(x,y) = \left(\frac{(2\times4)+(1\times-2)}{2+1}, \frac{(2\times-1)+(1\times-3)}{2+1}\right)$

$D(x,y) = \left(\frac{8-2}{3}, \frac{-2-3}{3}\right)$

$D(x,y) = \left(\frac{6}{3}, \frac{-5}{3}\right)$

$D(x,y) = \left(2, \frac{-5}{3}\right)$

23. Let α, β be the zeroes of the polynomial. then
    $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = -\frac{3}{2}$
    The quadratic polynomial
    $f(x) = (x-\alpha)(x-\beta)$
    = x^2 - $(\alpha+\beta)$x + $\alpha\beta$
    = $x^2 - \sqrt{2}x - \frac{3}{2}$
    Now, f(x) = $x^2 - \sqrt{2}x - \frac{3}{2}$
    $\Rightarrow f(x) = \frac{1}{2}\left(2x^2 - 2\sqrt{2}x - 3\right)$
    $\Rightarrow f(x) = \frac{1}{2}\left(2x^2 - 3\sqrt{2}x + \sqrt{2}x - 3\right)$
    $f(x) = \frac{1}{2}\{\sqrt{2}x(\sqrt{2}x-3) + (\sqrt{2}x-3)\}$
    $\Rightarrow f(x) = \frac{1}{2}(\sqrt{2}x-3)(\sqrt{2}x+1)$
    Now for f(x) = 0, we get
    $x = \frac{3}{\sqrt{2}}$ or, $x = -\frac{1}{\sqrt{2}}$
    Hence, the zeroes of f(x) are $\frac{3}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$.
24. We have to Construct a triangle similar to a given equilateral $\triangle$PQR with side 5 cm such that each of its side is $\frac{6}{7}$ of the corresponding sides of $\triangle$PQR. We write the steps of construction as follows:
    Steps of construction :
    i. Draw a line segment QR = 5 cm.
    ii. With Q as centre and radius = PQ = 5 cm, draw an arc.
    iii. With R as centre and radius = PR = 5 cm, draw another arc meeting the arc drawn in step 2 at the point P.
    iv. Join PQ and PR to obtain $\triangle$PQR.
    v. Below QR, construct an acute $\angle$RQX,
    vi. Along QX, mark off seven points Q_1,Q_2, ... Q_7 such that QQ_1 = Q_1Q_2 = Q_2Q_3....... = Q_6Q_7
    vii. Join Q_7R.
    viii. Draw Q_6R' | | Q_7R.
    ix. From R' draw R'P' | | RP.
    Hence, P'QR' is the required triangle.



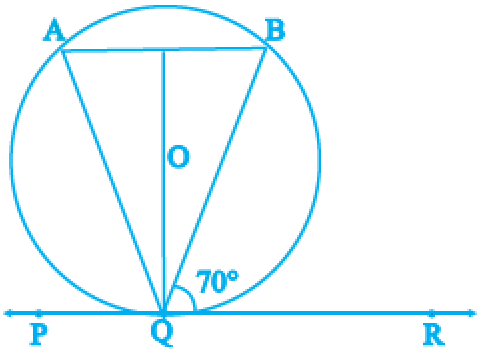
25. Let us first draw a right $\triangle$ABC
    Now, we have given that, tan $A = \frac{BC}{AB} = \frac{4}{3}$

Therefore, if BC = 4k, then AB = 3k, where k is any positive integer.
Now, by using the Pythagoras Theorem, we have
$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k$
So, AC = 5k
Now, we can write all the trigonometric ratios using their definitions
$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$
$\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$
Therefore, cot A = $\frac{1}{\tan A} = \frac{3}{4}, cosec A = \frac{1}{\sin A} = \frac{5}{4}$ and $\sec A = \frac{1}{\cos A} = \frac{5}{3}$

OR

Given that: $\sin\theta + \cos\theta = \sqrt{3}$
or $(\sin\theta + \cos\theta)^2 = 3$
or $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$
$2\sin\theta\cos\theta = 2$ [As $\sin^2\theta + \cos^2\theta = 1$]
or $\sin\theta\cos\theta = 1 = \sin^2\theta + \cos^2\theta$
LHS=1
RHS=$\frac{Sin\theta}{Cos\theta} + \frac{Cos\theta}{Sin\theta}$
Taking LCM
Substitute the value of LHS in RHS we get,
$\tan\theta + \cot\theta = 1$

26. Given,



PQR is a tangent
AB II PR
∠ BQR = 70
so ∠BAQ = ∠BQR = 70 (By Alternate segment theorem)
and ∠ABQ = ∠BQR = 70 (Alternate interior angles)
By summing all the angles
∠BAQ + ∠ABQ +∠AQB= 180
∠AQB = 180 - 70 -70
= 40

27. Let us find HCF of 396 and 231 using Euclid's division algorithm
$396 = 231 \times 1 + 165$
$231 = 165 \times 1 + 66$
$165 = 66 \times 2 + 33$
$66 = 33 \times 2 + 0$
$So\ HCF(396, 231) = 33$
So 33 is common factor of 396 and 231
and co-prime numbers have common factor of 1 only.
∴ The 396 and 231 are not co-prime.

28. Let Shefali's marks in Mathematics = x
Let Shefali's marks in English = 30 − x
If, she had got 2 marks more in Mathematics, her marks would be = x + 2

If, she had got 3 marks less in English, her marks in English would be = 30 – x – 3 = 27 – x
According to given condition:
⇒ (x + 2)(27 – x) = 210
$\Rightarrow 27x - x^2 + 54 - 2x = 210$
$\Rightarrow x^2 - 25x + 156 = 0$
Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,
We get a = 1, b = −25 and c = 156
Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{25 \pm \sqrt{(-25)^2 - 4(1)(156)}}{2 \times 1}$
$\Rightarrow \frac{25 \pm \sqrt{625 - 624}}{2}$
$\Rightarrow x = \frac{25 \pm \sqrt{1}}{2}$
$\Rightarrow x = \frac{25+1}{2}, \frac{25-1}{2}$
⇒ x = 13, 12
Therefore, Shefali's marks in Mathematics = 13 or 12
Shefali's marks in English = 30 – x = 30 – 13 = 17
Or Shefali's marks in English = 30 – x = 30 – 12 = 18
Therefore, her marks in Mathematics and English are (13, 17) or (12, 18).

OR

Let Rohan's present age be x years.
Then, his mother's age is (x + 26) years.
Rohan's age after 3 years = (x + 3) years.
After 3 years the age of Rohan's mother = (x + 26 + 3) years = (x + 29) years.
According to the question,
(x + 3)(x + 29) = 360.
$\Rightarrow x^2 + 32x - 273 = 0$.
This is the required quadratic equation.

29. We know that, if x = α is a zero of a polynomial, then x - α is a factor of f(x).
Since $\sqrt{2}$ and -$\sqrt{2}$ are zeros of f(x).
Therefore, (x - $\sqrt{2}$) { x + $\sqrt{2}$) = x^2 - 2 is a factor of f(x).
Let us now divide f(x) = $2x^4 - 3x^3 - 3x^2 + 6x - 2$ by g(x) = $x^2 - 2$
Using long division method, we obtain

$x^2 - 2 \overline{\big)\, 2x^4 - 3x^3 - 3x^2 + 6x - 2 \,}\big(2x^2 - 3x + 1$
$\quad 2x^4 \quad - 4x^2$
$\quad - \quad\quad + $
$\quad -3x^3 + x^2 + 6x - 2$
$\quad -3x^3 \quad + 6x$
$\quad + \quad\quad -$
$\quad x^2 - 2$
$\quad x^2 - 2$
$\quad - \quad +$
$\quad 0$

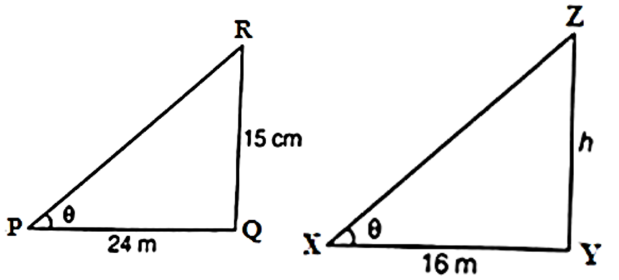
Thus, Quotient = $2x^2 - 3x + 1$ and Remainder = 0.
By division algorithm, we have
f(x) = Quotient × Divisor + Remainder
$\Rightarrow f(x) = (x^2 - 2)(2x^2 - 3x + 1) + 0$
$\Rightarrow \quad f(x) = (x - \sqrt{2})(x + \sqrt{2})\left(2x^2 - 2x - x + 1\right)$
$\Rightarrow \quad f(x) = (x - \sqrt{2})(x + \sqrt{2})\{2x(x-1) - 1(x-1)\}$
$\Rightarrow \quad f(x) = (x - \sqrt{2})(x + \sqrt{2})(x-1)(2x-1)$

On equating factors $x-\sqrt{2}, x+\sqrt{2}$, x - 1 and 2x - 1 to zero, we get x = $\sqrt{2},-\sqrt{2}, 1, \frac{1}{2}$.

Hence, the zeros of the given polynomial are $\sqrt{2},-\sqrt{2}, 1$ and $\frac{1}{2}$.

30. Given,

Let



QR = 15 m (height of tower)

PQ = 24 m (shadow of tower)

At that time $\angle$RPQ = θ

Again, let YZ = h be a telephone pole and its shadow XY = 16 m.

The same time $\angle$YXZ = θ

Here, ABC and $\triangle$DEF both are right angles triangles.

In $\triangle$PQR and $\triangle$XYZ,

$\angle$RPQ = $\angle$YXZ = θ

$\angle$Q = $\angle$Y [each 90°]

$\therefore$ $\triangle$PQR $\cong$ $\triangle$XYZ [by AAA similarity criterion]

Then,

$\frac{PQ}{XY}=\frac{QR}{YZ}$

$\Rightarrow \frac{24}{16}=\frac{15}{h}$

$\therefore \mathrm{h}=\frac{15 \times 16}{24}=10$

Hence, the height of the telephone pole is 10 m.

OR

Given: $\triangle ABC$ in which $AD \perp BC$ and BD = 3CD.

To Prove: $2AB^2 = 2AC^2 + BC^2$.

Proof: We have BD = 3CD.

$\therefore$ BC = BD + CD = 3CD + CD = 4CD

$\Rightarrow \quad CD=\frac{1}{4} BC$ ...(i)

In $\triangle ADB, \angle ADB=90^{\circ}$.

Thus, by using Pythagoras Theorem we can write,

$\therefore AB^2 = AD^2 + BD^2$ ...(ii)

In $\triangle ADC, \angle ADC=90^{\circ}$

Thus, by using Pythagoras Theorem we can write,

$\therefore AC^2 = AD^2 + CD^2$. ...(iii)

On subtracting (iii) form (ii), we get

$AB^2 - AC^2 = BD^2 - CD^2$

$AB^2 - AC^2 = [(3CD)^2 - (CD)^2] = 8CD^2$ [$\because$ BD = 3CD]

$AB^2 - AC^2 = 8 \times \frac{1}{16} BC^2=\frac{1}{2} BC^2$ [using (i)].

$\therefore 2AB^2 - 2AC^2 = BC^2$

Hence, $2AB^2 = 2AC^2 + BC^2$

Hence Proved

31. The total numbers of integers between 0 and 100 = 99

Numbers divisible by 7 are 7,14,21,28,35,42,49,56,63,70,77,84,91,98

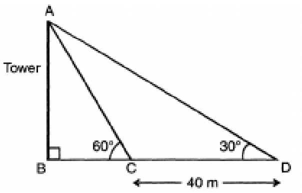
(i) Let E_1 be the event of getting a integer divisible by 7

$\therefore$ Number of favourable outcomes = 14

P(E_1) = $\frac{14}{99}$

(ii) Let E_2 be an event of getting a number not divisible by 7

$\therefore$ P(E_2)=1-P(E_1)= $1 - \frac{14}{99} = \frac{85}{99}$

32. 

In $\triangle ABC$, $\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \quad AB = \sqrt{3}BC$ .....(i)

In $\triangle ABD$,

$\tan 30^\circ = \frac{AB}{BC+40}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BC+40} = \frac{\sqrt{3}BC}{BC+40}$

3BC = BC + 40

BC = 20, Hence from (i) we get

AB = 20√3 = 20 $\times$ 1.73 = 34.6 meter

33. First, we will convert the graph given into tabular form as shown below:

Class interval	**Frequency (f_i)**	**Mid value (x_i)**	**$f_i x_i$**	**Cumulative Frequency**
1 – 4	6	2.5	15	6
4 – 7	30	5.5	165	36
7 – 10	40	8.5	340	76
10 – 13	16	11.5	184	92
13 – 16	4	14.5	58	96
16 – 19	4	17.5	70	100
	N = $\sum f_i$ = 100		$\Sigma f_i x_i = 832$	

i. N = 100

Mean $= \frac{\Sigma f_i x_i}{N} = \frac{832}{100} = 8.32$

ii. $\frac{N}{2} = \frac{100}{2} = 50$

The cumulative frequency just greater than $\frac{N}{2}$ is 76, then the median class is 7 - 10 such that

$l = 7, h = 10 - 7 = 3, f = 40, F = 36$

Median $= l + \frac{\frac{N}{2} - F}{f} \times h$

$= 7 + \frac{50-36}{40} \times 3$

$= 7 + \frac{42}{40} = 7 + 1.05 = 8.05$

iii. Mode = 3 Median - 2 Mean

= 3$\times$8.05 - 2$\times$8.32 = 7.51

34. Suppose, the digit at units and tens place of the given number be x and y respectively.

$\therefore$ the number is $10y + x$

After interchanging the digits, the number becomes $10x + y$

Given: The sum of the numbers obtained by interchanging the digits and the original number is 66.

Thus, $(10x + y) + (10y + x) = 66$

$\Rightarrow 10x + y + 10y + x = 66$

$\Rightarrow 11x + 11y = 66$

$\Rightarrow 11(x + y) = 66$

$\Rightarrow x + y = \frac{66}{11}$

$\Rightarrow x + y = 6$ .....(i)

Also given, the two digits of the number are differing by 2.

$\therefore$ we have $x - y = \pm 2$....(ii)

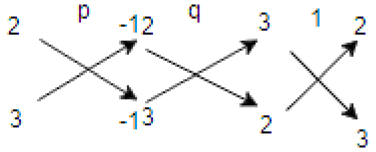
So, we have two systems of simultaneous equations,
$x - y = 2,\ x + y = 6$
$x - y = -2,\ x + y = 6$
Here x and y are unknowns. We have to solve the above systems of equations for x and y.

1. First, we solve the system
   $x - y = 2$
   x + y = 6
   Adding the two equations,
   $\Rightarrow (x - y) + (x + y) = 2 + 6$
   $\Rightarrow x - y + x + y = 8$
   $\Rightarrow 2x = 8$
   $\Rightarrow x = \frac{8}{2}$
   $\Rightarrow x = 4$
   Substituting the value of x in the first equation, we have
   $4 - y = 2$
   $\Rightarrow y = 4 - 2$
   $\Rightarrow y = 2$
   Hence, the number is 10 $\times$ 2 + 4 = 24
2. Now, we solve the system
   $x - y = -2$
   $x + y = 6$
   Adding the two equations, we have
   $(x - y) + (x + y) = -2 + 6$
   $\Rightarrow x - y + x + y = 4$
   $\Rightarrow 2x = 4$
   $\Rightarrow x = \frac{4}{2}$
   $\Rightarrow$ x = 2
   Substituting the value of x in the first equation,
   $\Rightarrow 2 - y = -2$
   $\Rightarrow y = 2 + 2$
   $\Rightarrow$ y = 4
   Hence, the number is 10 $\times$ 4 + 2 = 42

   Thus, the two numbers are 24 and 42.

OR

$\frac{1}{2x} + \frac{1}{3y} = 2$ ... (1)
$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$ ...(2)
Let $\frac{1}{x}$ = p and $\frac{1}{y}$ = q
Putting this in equation (1) and (2), we get
$\frac{p}{2} + \frac{q}{3} = 2$ and $\frac{p}{3} + \frac{q}{2} = \frac{13}{6}$
$\Rightarrow$ 3p + 2q = 12 and 6(2p +3q) = 13 (6)
$\Rightarrow$ 3p + 2q = 12 and 2p + 3q = 13
$\Rightarrow$ 3p + 2q - 12 = 0 .... (3) and
2p + 3q - 13 = 0 .... (4)



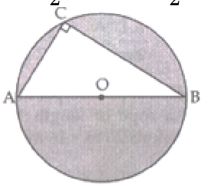
$\frac{p}{2(-13)-3(-12)} = \frac{q}{(-12)2-(-13)3} = \frac{1}{3\times3-2\times2}$
$\Rightarrow \frac{p}{-26+36} = \frac{q}{-24+39} = \frac{1}{9-4}$
$\Rightarrow \frac{p}{10} = \frac{q}{15} = \frac{1}{5} \Rightarrow \frac{p}{10} = \frac{1}{5}$ and $\frac{q}{15} = \frac{1}{5}$

$\Rightarrow$ p = 2 and q = 3

But $\frac{1}{x}$ = p and $\frac{1}{y}$ = q

Putting value of p and q in this we get,

x = $\frac{1}{2}$ and y = $\frac{1}{2}$

35. 

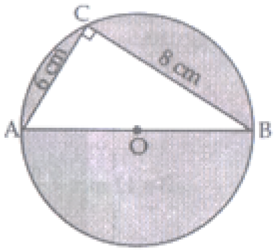
Identify the figure as a circle, and a right-angled triangle (and semicircle, segment also) since AOB is diameter and angle in semicircle is 90^o.

Therefore, $\angle$C = 90^o

In right-angled $\triangle$ABC,

b = base = BC = 8 cm

a = altitude = AC = 6 cm



Therefore,by Pythagoras theorem in right $\triangle$ABC,

$AB^2 = BC^2 + AC^2$

$= 8^2 + 6^2 = 64 + 36$

$\Rightarrow AB^2$ = 100 cm

$\Rightarrow$ AB = 10 cm

Therefore, r = $\frac{10}{2}$ = 5 cm

Therefore, Area of shaded region = Area of circle – Area of right $\triangle$ABC

= $\pi r^2 - \frac{1}{2}$ Base × Alt.

= $3.14 \times 5 \times 5 - \frac{1}{2} \times 8 \times 6$

= $3.14 \times 25 - 8 \times 3$ = (78.50 – 24) cm^2 = 54.50 cm^2

Therefore, Area of shaded region = 54.50 cm^2

36. Let h is height of big building,here as per the diagram.

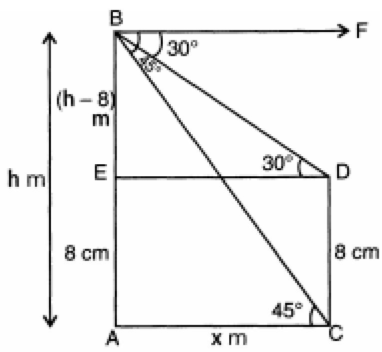
AE = CD = 8 m (Given)

BE = AB-AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$



In $\triangle$ACB, $\angle A = 90^\circ$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$
$\Rightarrow \quad x = \sqrt{3}(h-8)$.(ii)
From (i) and (ii), we get
$h = \sqrt{3}h - 8\sqrt{3}$
h(√3 - 1) = 8√3
h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84) = 18.92m$
Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.