Solution

Class 10 - Mathematics

2020-2021 - Paper-6

Part-A

1. The given rational number can be expressed as $\frac{6}{15} = \frac{2\times3}{3\times5} = \frac{2}{5} = \frac{2\times2}{5\times2} = \frac{4}{10} = 0.4$

OR

140 = 2 × 2 × 5 × 7 = $2^2 \times 5 \times 7$

2. $(x-1)^2 + 2(x+1) = 0$
$x^2 - 2x + 1 + 2x + 2 = 0$
$x^2 + 3 = 0$
No, since the equation is simplified to $x^2 + 3 = 0$ whose discriminant is less than zero.

3. Condition for coincident lines is given by:
$a_1/a_2 = b_1/b_2 = c_1/c_2$;
Given pair of linear equations is
- 2x - 3y - 1 = 0 and 4x + 6y + 2 = 0;
Comparing with standard form we have;
$a_1 = -2$, $b_1 = -3$, $c_1 = -1$;
And $a_2 = 4$, $b_2 = 6$, $c_2 = 2$;
$a_1/a_2 = -2/4 = -1/2$
$b_1/b_2 = -3/6 = -1/2$
$c_1/c_2 = -1/2$
Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e., coincident lines
Hence, the given pair of linear equations is coincident.

4. Two parallel tangents can exist at the two ends of the diameter of the circle. Therefore, the distance between the two parallel tangents will be equal to the diameter of the circle. In the problem the radius of the circle is given as 5 cm. Therefore,
Diameter = 5 × 2
Diameter = 10 cm
Hence, the distance between the two parallel tangents is 10 cm.

5. a = -1, $d = \frac{1}{2}$
First term = a = -1
Second term = -1 + d $= -1 + \frac{1}{2} = -\frac{1}{2}$
Third term $= -\frac{1}{2} + d = -\frac{1}{2} + \frac{1}{2} = 0$
Fourth term = 0 + d $= 0 + \frac{1}{2} = \frac{1}{2}$
Hence, the first four terms of the given AP are -1, $-\frac{1}{2}, 0, \frac{1}{2}$.

OR

Here $a = 2$;
$d = 7 - 2 = 5$.
So, $a_{10} = a + 9d = 2 + 9 \times 5 = 47$

6. We have $a_1 = 1$, $a_2 = 1$, $a_3 = 2$ and $a_4 = 2$
$a_2 - a_1 = 0$
$a_3 - a_2 = 1$
Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

7. Given $(x-\sqrt{2})^2 - \sqrt{2}(x+1) = 0$
Simplifying the above equation,
⇒ $x^2 - 2\sqrt{2}x + 2 - \sqrt{2}x - \sqrt{2} = 0$
⇒ $x^2 - \sqrt{2}(2+1)x + (2-\sqrt{2}) = 0$

$\therefore x^2 - 3\sqrt{2}x + (2 - \sqrt{2}) = 0$
$\therefore$ D = b^2 - 4ac
$= (-3\sqrt{2})^2 - 4(1)(2 - \sqrt{2})$
$= 18 - 8 + 4\sqrt{2} > 0$
Hence, the roots are real and distinct.

OR

The equation (x - 1) (x + 2) + 2 = 0 has two real and distinct roots.
Simplifying the above equation, we have

x^2 - x + 2x - 2 + 2 = 0

$\therefore$ x^2 + x = 0

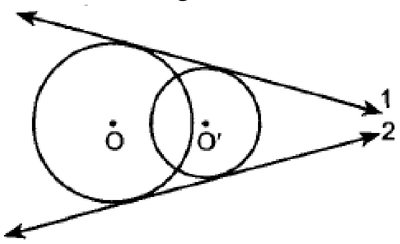
$\therefore$ D = b^2 - 4ac

= 12 - 4(1)(0)

= 1 - 0 > 0

Hence, the roots are real and distinct.

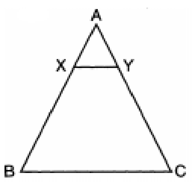
8. 2 common tangents can be drawn to two circles intersecting at two distinct points.



9. OP $\perp$ PR [$\because$ Tangent and radius are $\perp$ to each other at the point of contact]
$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$
$OP = OQ$ [By isosceles triangle's property]
$\angle$OPQ = $\angle OQP = 40^\circ$
In $\triangle$OPQ,
$\Rightarrow \angle$O + $\angle$P + $\angle$Q = 180°
$\Rightarrow \angle O + 40^\circ + 40^\circ = 180^\circ$
$\angle O = 180^\circ - 80^\circ = 100^\circ$.

OR

In Fig , OA is the radius and AC is the tangent from the external point C.
Therefore, $\angle OAC = 90^\circ$ (Theorem:Radius and tangent are always perpendicular to each other at the point of contact)
$\angle BOC = \angle OAC + \angle ACO$ (Exterior angle property)
$\Rightarrow 130^\circ = 90^\circ + \angle ACO$
$\Rightarrow \angle ACO = 130^\circ - 90^\circ = 40^\circ$

10. 

$\frac{AX}{XB} = \frac{3}{4} \ldots . (i)$
$\frac{AY}{CY} = \frac{5}{9} \ldots . (ii)$
From eqn (i) and (ii)
$\frac{AX}{XB} \neq \frac{AY}{YC}$
So XY and BC are not parallel

11. The two -digit numbers divisible by 3 start from 12,15,18,21,...,99
Here,
$a = 12$
$d = 3$
$a_n = a + (n-1)d$
$\Rightarrow 99 = 12 + (n-1)(3)$

$\Rightarrow 99 = 12 + 3n - 3$

$\Rightarrow 90 = 3n$

$\Rightarrow$ n=30

Thus, 30 two-digit numbers are divisible by 3.

12. L.H.S. = $(\sin^4 \theta - \cos^4 \theta + 1) \operatorname{cosec}^2 \theta$

$= [(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) + 1] \operatorname{cosec}^2 \theta$

$= (\sin^2 \theta - \cos^2 \theta + 1) \operatorname{cosec}^2 \theta$ [Because $\sin^2 \theta + \cos^2 \theta = 1$]

$= 2\sin^2 \theta \operatorname{cosec}^2 \theta$ [Because $1 - \cos^2 \theta = \sin^2 \theta$]

= 2 = RHS

13. We have ,

$\frac{cosec\ 32^\circ}{\sec 58^\circ}$

$= \frac{cosec(90^\circ - 58^\circ)}{\sec 58^\circ}$

$= \frac{\sec 58^\circ}{\sec 58^\circ}$ [Since, Cosec (90° - A) = SecA] .

=1

14. Volume of the cube = $(5 \times 5 \times 1 + 2)$ = 27 cm^3

Length of edge of the cube = $\sqrt[3]{\text{volume}} = \sqrt[3]{27}$= 3 cm

15. Given AP = $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2} \ldots$

1st term = $\frac{3}{2}$

Common difference = $\frac{1}{2} - \frac{3}{2}$ = -1

16. Since, P(E) + P (not E) = 1

P (not E) = 1 - P(E) = 1 - 0.05 = 0.95

OR

Number of total outcomes = 36

When product of numbers appearing on them is less than 9, then possible ways are (1, 6), (1, 5), (1, 4), (1, 3), (1, 2), (1, 1), (2, 2), (2, 3), (2, 4), (3, 2), (4, 2), (4, 1), (3, 1), (5, 1), (6, 1) and (2, 1).

Number of possible ways = 16

Probability $= \frac{\text{no. of favorable outcomes}}{\text{Total outcomes}}$

$\therefore$ Required probability = $\frac{16}{36} = \frac{4}{9}$

17. i. (c) It can be observed that Niharika posted the green flag at 5th position flower of the distance AD i.e., 5×5 = 25m from the starting point of 2nd line.
   ii. (d) The coordinates of the Green flag are (2, 25).
   iii. (b) The point at which Rashmi should post her blue flag is the mid-point of the line joining these points.
   Let this point be A(x, y)
   Now by midpoint formula,
   $(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$
   $x = \frac{2+8}{2} = 5$
   $y = \frac{25+20}{2} = 22.5$
   Hence, A(x, y) = (5, 22.5)
   Therefore, Rashmi should post her blue flag at 22.5 m on the 5th line.
   iv. (a) According to the distance formula,
   Distance between these flags by using the distance formula, D
   $= [(8 - 2)^2 + (25 - 20)^2]^{1/2} = (36 + 25)^{1/2} = \sqrt{61}m$
   v. Preet posted a red flag at the distance of 4th flower position of AD i.e., 4×5 = 20m from the starting point of the 8th line. Therefore, the coordinates of this point R are (8, 20).

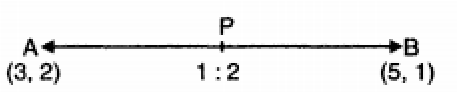
18. i. (b) 60 m^2
   ii. (a) 2.5 m
   iii. (b) 6 m
   iv. (c) 3.75 m^2
   v. (a) 21.6 m^2

19. i. (b) Centered at the class marks of the classes
    ii. (c) 80
    iii. (a) 44.7
    iv. (b) Classmark
    v. (d) 42
20. i. (a) 77 m^2
    ii. (d) $\pi r^2 h+\frac{2}{3}\pi r^3$
    iii. (c) 56.57 m^3
    iv. (d) 628.57 m^2
    v. (c) 1:1

Part-B

21. Let a be of the form $n^2 - 1$, where n is odd
    i.e. n = 2k + 1, where k is an integer.
    $\Rightarrow$ a = $(2k + 1)^2$ -1
    $\Rightarrow$ a = $4k^2$ + 1 + 4k-1
    $\Rightarrow$ a = $4k^2$ + 4k = 4k(k + 1)
    Now, Either k is even or k+1 is even.
    In former case, k = 2r $\Rightarrow$ a = 8r(2r + 1)
    In later case, k + 1 = 2r $\Rightarrow$ a = 8r(2r - 1)
    Thus, in any case n^2 - 1 is divisible by 8.
22. (a, b), (-a, -b)
    Required distance
    $=\sqrt{(-a-a)^2+(-b-b)^2}$
    $=\sqrt{(-2a)^2+(-2b)^2}$
    $=\sqrt{4a^2+4b^2}$
    $=\sqrt{4(a^2+b^2)}$
    $=2\sqrt{a^2+b^2}$

OR

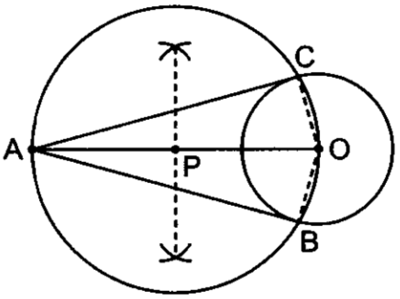


Since, the line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2
Therefore, according to the section formula,
$x=\frac{mx_2+nx_1}{m+n}=\frac{1\times5+2\times3}{1+2}=\frac{11}{3}$
$and,\ y=\frac{my_2+ny_1}{m+n}=\frac{1\times1+2\times2}{1+2}=\frac{5}{3}$
$\implies (x,y)=\left(\frac{11}{3},\frac{5}{3}\right)$ lies on $3x-18y+k=0$
Therefore, these points satisfy equation of given line.
Hence, $3\times\frac{11}{3}-18\times\frac{5}{3}+k=0$
$\Rightarrow 11-30+k=0$
$\Rightarrow$ k = 19,
Hence required value of k =19

23. Given quadratic equation: x^2 - 3
    Recall the identity $a^2 - b^2$ = (a – b)(a + b). Using it,
    we can write:
    $x^2-3=(x-\sqrt{3})(x+\sqrt{3})$
    So, the value of $x^2 - 3$ is zero when x = $\sqrt{3}$ or x = $-\sqrt{3}$
    Therefore, the zeroes of x^2-3 are $\sqrt{3}$ and $-\sqrt{3}$

Now, the sum of zeroes = $\sqrt{3} - \sqrt{3} = 0 = \frac{-(\text{ Coefficient of } x)}{\text{Coefficient of } x^2}$

and the product of zeroes = $(\sqrt{3})(-\sqrt{3}) = -3 = \frac{-3}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

24. 

Steps of Construction:

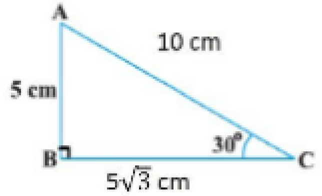
i. Let O be the center and A is any point outside it. Join AO and bisect it. Let P be the mid-point of AO.
ii. Taking P as centre and PO as radius, draw a circle. Let it intersect the given circle at the points B and C.
iii. Join AB and AC. AB and AC are the required tangents as shown in given figure.
Note: AC $\perp$ OC; AB $\perp$ OB

25. $\sec 4A = cosec\,(A - 20^\circ)$
$\Rightarrow cosec\,(90^\circ - 4A) = cosec\,(A - 20^\circ) \because cosec\,(90^\circ - \theta) = \sec\theta$
$\Rightarrow 90^\circ - 4A = A - 20^\circ \quad \because 90^\circ - 4A$ and $A - 20^\circ$ are both acute angles
$\Rightarrow 5A = 110^\circ \Rightarrow A = \frac{110^\circ}{5} = 22^\circ$

OR

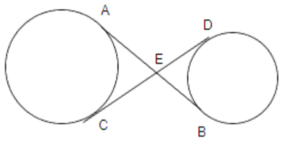
Given AB = 5 cm

$\angle$ACB = 30^o



According to diagram,
tan C = $\frac{side\ opposite\ to\ angle\ C}{side\ adjacent\ to\ angle\ C}$
tan 30^o = $\frac{AB}{BC}$
$\frac{1}{\sqrt{3}}$ = $\frac{5}{BC}$
BC = $5\sqrt{3}$ cm
sin C = $\frac{side\ of\ angle\ C}{hypotenuse}$
sin 30^o = $\frac{AB}{AC}$
$\frac{1}{2}$ = $\frac{5}{AC}$
AC = 10 cm.

26. Common tangents AB and CD to two circles intersect at E.
As we know that , the tangents drawn from an external point to a circle are equal in length.
$\therefore$ EA = EC....(i)
and EB = ED...(ii)
On adding Eqs (i) and (ii), we get
EA + EB = EC + ED
$\Rightarrow$ AB = CD



27. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.
That is, we can find coprime numbers a and b (b ≠ 0) such that $5 - \sqrt{3} = \frac{a}{b}$
Therefore, $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$
Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.
But this contradicts the fact that $\sqrt{3}$ is irrational
This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.
So, we conclude that $5 - \sqrt{3}$ is irrational.

28. Let the speed of the passenger train be x km/hr. Then, the speed of express train will be (x + 11)km/hr
Time taken by the express train to cover 132 km $= \frac{132}{x+11} hrs$
Time taken by the passenger train to cover 132 km $= \frac{132}{x} hrs$
According to the question ;
$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$
$\Rightarrow \frac{132(x+11)-132x}{x(x+11)} = 1$
$\Rightarrow$ 132x - 132x + 1452 = x(x +11)
$\Rightarrow$ 1452 = x^2 + 11x
$\Rightarrow$ x^2 + 11x - 1452 = 0 , This is the required quadratic equation.
Now, x^2 + 11x - 1452 = 0
$\Rightarrow x^2 + 44x - 33x - 1452 = 0$
$\Rightarrow x(x+44) - 33(x+44) = 0$
$\Rightarrow (x-33)(x+44) = 0$
$\therefore x = 33$ or $x = -44$ (Rejected)
Thus speed of passenger train is 33km/hour and that of express train =44 km/hour.

OR

Given, $2x^2$ - 5x + 3 = 0
$\Rightarrow \quad x^2 - \frac{5}{2}x + \frac{3}{2} = 0$ [Dividing throughout by 2]
$\Rightarrow \quad x^2 - \frac{5}{2}x = -\frac{3}{2}$ [Shifting the constant term on RHS]
$\Rightarrow \quad x^2 - \left(\frac{5}{2}\right)x + \left(\frac{5}{4}\right)^2 = \left(\frac{5}{4}\right)^2 - \frac{3}{2} \left[\text{ Adding } \left(\frac{1}{2}\ of\ \text{ Coeff. of } x\right)^2 \text{ on both sides}\right]$
$\Rightarrow x^2 - 2 \times \frac{5}{4}x \times (\frac{5}{4})^2 = (\frac{5}{4})^2 - \frac{3}{2}$
$\Rightarrow \quad \left(x - \frac{5}{4}\right)^2 = \frac{25}{16} - \frac{3}{2}$
$\Rightarrow \quad \left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$
$\Rightarrow \quad x - \frac{5}{4} = \pm\frac{1}{4} \Rightarrow x = \frac{5}{4} \pm \frac{1}{4}$
$\Rightarrow \quad x = \frac{5}{4} + \frac{1}{4} = \frac{6}{4}$ or, $x = \frac{5}{4} - \frac{1}{4} = \frac{4}{4} = 1 \Rightarrow x = \frac{3}{2}$ or, $x = 1$
Therefore, the roots are $\frac{3}{2}$ and 1.

29. We will find zeroes of p(x) which are not the zeroes of q(x) by using Factor theorem and Euclid's division algorithm.
By factor theorem if q(x) is a factor of p(x), then r(x) must be zero.
p(x) = $x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$
q(x) = $x^3 + 2x^2 + a$

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
x^3 + 2x^2 + a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\
 & \underline{-}x^5 \underline{+} 2x^4 \quad \underline{+} ax^2 \\
 & -3x^4 - 4x^3 - ax^2 + 3x^2 + 3x + b \\
 & -3x^4 - 6x^3 \quad - 3ax \\
 & + \quad + \quad + \\
 & 2x^3 - ax^2 + 3x^2 + 3ax + 3x + b \\
 & 2x^3 \quad \underline{+} 4x^2 \quad \underline{+} 2a \\
 & - \\
 & -ax^2 - x^2 + 3ax + 3x - 2a + b
\end{array}
$$

So, by factor theorem remainder must be zero i.e.,
r(x) = 0

Value of r(x) is - $ax^2 - x^2 + 3ax + 3x - 2a + b$.

$- ax^2 - x^2 + 3ax + 3x - 2a + b = 0x^2 + 0x + 0$

$\Rightarrow -(a + 1)x^2 + (3a + 3)x + (b - 2a) = 0x^2 + 0x + 0$

Comparing the coefficients of x^2, x and constant. on both sides, we get

-(a + 1) = 0 and 3a + 3 = 0 and b – 2a = 0

a + 1 = 0

a = -1

and 3a + 3 = 0

3a = - 3

a = -1

Put a = -1 in b - 2a = 0

Then b – 2(-1) = 0

⇒ b + 2 = 0

⇒ b = -2

For a = -1 and b = -2, zeroes of q(x) will be zeroes of p(x).

For zeroes of p(x), p(x) = 0

$\Rightarrow (x^3 + 2x^2 + a)(x^2 - 3x + 2) = 0$ [∵ a = -1]

$\Rightarrow [x^3 + 2x^2 - 1][x^2 - 2x - 1x + 2] = 0$

$\Rightarrow (x^3 + 2x^2 - 1)[x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x^3 + 2x^2 - 1)(x - 2)(x - 1) = 0$

⇒ (x – 2) = 0 and (x – 1) = 0

⇒ x = 2 and x = 1

we find that q(1) = 1 + 2 - 1 = 2 $\neq$ 0 and q(2) = 8 + 8 - 1 = 15 $\neq$ 0.

So, the zeros the zeros of quotient x^2-3x+2 are not zeros of q(x)

Hence, x = 2 and 1 are not the zeroes of q(x).

30. Using Pythagoras theorem in $\triangle$ABM, we obtain

$AM^2 + MB^2 = AB^2$ ...(1)

Applying Pythagoras theorem in $\triangle$AMC, we obtain

$AM^2 + MC^2 = AC^2$ ...(2)

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

$$2\left(\mathrm{AM}^2+\mathrm{MD}^2\right)+\left(\frac{\mathrm{BC}}{2}\right)^2+\left(\frac{\mathrm{BC}}{2}\right)^2+2\mathrm{M}\left(\frac{-\mathrm{BC}}{2}+\frac{\mathrm{BC}}{2}\right)=\mathrm{AB}^2+\mathrm{AC}^2$$

$$2\mathrm{AD}^2+\frac{\mathrm{BC}^2}{2}=\mathrm{AB}^2+\mathrm{AC}^2$$

or $\mathrm{AC}^2+\mathrm{AB}^2=2\mathrm{AD}^2+\frac{1}{2}\mathrm{BC}^2$

Hence proved

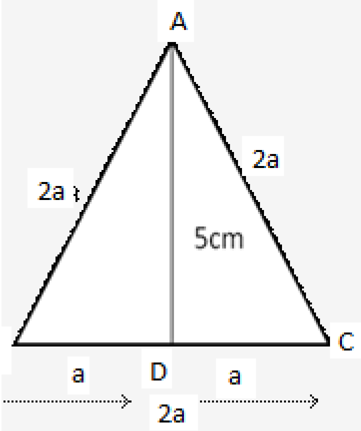
OR

In an equilateral triangle all sides are equal.

Then, AB=BC=AC=2a units

Construction :- Draw an altitude AD $\perp$BC

Given BC=2a. Then, BD=a [as in an equilateral triangle median and altitude coincide with each other]



In $\triangle$ABD,

$\angle$ADB=90^{o} (by pythagoras theorem)

$AB^2 = AD^2 + BD^2$

$(AD)^2=(AB^2 - BD^2)$

$=[(2a)^2-(a)^2]$

$=(4a^2-a^2) = 3a^2$

AD=$\sqrt{3a^2}$unit =a$\sqrt{3}$ units

Hence, length of each altitude in an equilateral triangle with side 2a is a$\sqrt{3}$ units.

31. The total numbers of integers between 0 and 100 = 99

Numbers divisible by 7 are 7,14,21,28,35,42,49,56,63,70,77,84,91,98

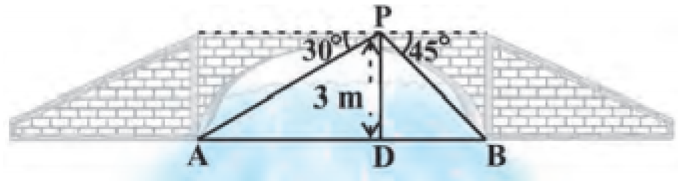
(i) Let E_1 be the event of getting a integer divisible by 7

$\therefore$ Number of favourable outcomes = 14

P(E_1) = $\frac{14}{99}$

(ii) Let E_2 be an event of getting a number not divisible by 7

$\therefore$ P(E_2)=1-P(E_1)= $1-\frac{14}{99}=\frac{85}{99}$

32. 

In the above-given fig, A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river.

P is a point on the bridge at a height of 3 m, i.e.,

DP = 3 m. Here, we are interested to determine the width of the river, which is the length of the side AB of the $\triangle$ APB.

Now, AB = AD + DB

In right $\triangle$ APD, $\angle$ A = 30°

So, tan 30° = $\frac{PD}{AD}$

i.e., $\frac{1}{\sqrt{3}}=\frac{3}{AD}$ or AD = $3\sqrt{3}$ m

Also, in right $\triangle$ PBD, $\angle$ B = 45°. So, BD = PD = 3 m

Now, AB = BD + AD = 3 + $3\sqrt{3}$ = $3(1+\sqrt{3})$m

Therefore, the width of the river is $3(\sqrt{3}+1)$m

33. First, we will convert the graph into tabular form given below:

Monthly consumption (in units)	**Number of consumers (f_i)**	**Class mark (x_i)**	**$d_i = x_i - 135$**	$u_i=\frac{x_i-135}{5}$	**f_iu_i**	**Cumulative Frequency**
65-85	4	75	–60	–3	–12	4
85-105	5	95	–40	–2	–10	9
105-125	13	115	–20	–1	–13	22

125-145	20	135	0	0	0	42
145-165	14	155	20	1	14	56
165-185	8	175	40	2	16	64
185-205	4	195	60	3	12	68
Total	$\sum f_i = 68$				$\sum f_i u_i = 7$	

i. Let a = 135.
Now, h = 20
Using the step-deviation method,
$Mean, \overline{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h = 135 + \left(\frac{7}{68}\right) \times 20$
$= 135 + \frac{35}{17}$ = 135 + 2.05 = 137.05

ii. Now, N = 68
So, $\frac{N}{2} = \frac{68}{2} = 34$
This observation lies in class 125-145.
Therefore, 125-145 is the median class.
So, l = 125, CF = 22, f = 20
$\therefore Median = l + \left(\frac{\frac{N}{2} - CF}{f}\right) \times h$
$= 125 + \left(\frac{34-22}{20}\right) \times 20$ = 125 + 12 = 137

iii. Mode = 3 Median - 2 Mean
= 3×137 - 2×137.05 = 136.9

34. Let the time taken by 1 woman alone to finish to embroidery be x days
and the time taken by 1 man alone to finish to embroidery be y days. Then
1 woman's 1 day's work $= \frac{1}{x}$ and 1 man's 1 day's work $= \frac{1}{y}$
$\therefore$ 2 women's 1 day's work $= \frac{2}{x}$ and 5 man's 1 day's work $= \frac{5}{y}$
$\because$ 2 women and 5 men can together finish a piece of embroidery in 4 days.
$\therefore\ 4\left(\frac{2}{x} + \frac{5}{y}\right) = 1 \Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$ ...(1)
Again, 3 women and 6 men can together finish a piece of embroidery in 3 day
$\therefore\ 3\left(\frac{3}{x} + \frac{6}{y}\right) = 1 \Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$ ...(2)
Put $\frac{1}{x} = u$ ...(3) and $\frac{1}{y} = v$ ...(4)
The equation (1) and (2) can be rewritten as
$2u + 5v = \frac{1}{4}$ ...(5)
$3u + 6v = \frac{1}{3}$ ...(6)
Multiplying equation (5) by 3 and equation (6) by 2, we get
$6u + 15v = \frac{3}{4}$ ...(7)
$6u + 12v = \frac{2}{3}$ ...(8)
Subtracting equation (8) from equation (7), we get
$3v = \frac{3}{4} - \frac{2}{3} = \frac{1}{12} \Rightarrow v = \frac{1}{36}$ ...(9)
Substituting this value of v in equation (5), we get $2u + 5\left(\frac{1}{36}\right) = \frac{1}{4}$
$\Rightarrow\ 2u + \frac{5}{36} = \frac{1}{4} \Rightarrow 2u = \frac{1}{4} - \frac{5}{36}$
$\Rightarrow\ 2u = \frac{9}{36} - \frac{5}{36} \Rightarrow 2u = \frac{4}{36} = \frac{1}{9} \Rightarrow u = \frac{1}{18}$ ...(10)
From equation (3) and equation (10), we get $\frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$
From equation (4) and equation (9), we get $\frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$
Hence, the time taken by 1 women alone to finish the embroidery is 18 days
and the time taken by 1 man alone to finish the embroidery is 36 days.
Verification, Substituting x = 18, y = 36, We find the that both the equations
(1) and (2) are satisfied as shown below:

$\frac{2}{x}+\frac{5}{y}=\frac{2}{18}+\frac{5}{36}=\frac{1}{9}+\frac{5}{36}=\frac{1}{4}; \frac{3}{x}+\frac{6}{y}=\frac{3}{18}+\frac{6}{36}=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$

This verifies the solution.

OR

The given pair of equations is

$\frac{7x-2y}{xy}=5$ .... (1)

$\frac{8x+7y}{xy}=15$ .... (2)

$\Rightarrow \quad \frac{7x}{xy}-\frac{2y}{xy}=5$........ (3)

and $\frac{8x}{xy}+\frac{7y}{xy}=15$......... (4)

$\Rightarrow \quad \frac{7}{y}-\frac{2}{x}=5$ .... (5)

and $\frac{8}{y}+\frac{7}{x}=15$ .... (6)

Put $\frac{1}{x}=u$ .... (7)

And $\frac{1}{y}=v$ ...... (8)

Then the equation (5) and (6) can be rewritten as

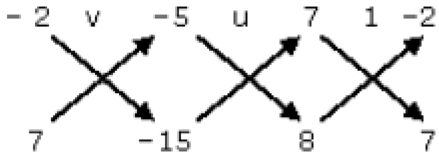
7v - 2u = 5 .... (9)

8v + 7u = 15 .... (10)

$\Rightarrow \quad 7v-2u-5=0$ .... (11)

8v + 7u - 15 = 0 .... (12)

To solve the equations by the cross multiplication method, we draw the diagram below:



Then,

$\Rightarrow \quad \frac{v}{(-2)(-15)-(7)(-5)}=\frac{u}{(-5)(8)-(-15)(7)}=\frac{1}{(7)(7)-(8)(-2)}$

$\Rightarrow \quad \frac{v}{30+35}=\frac{u}{-40+105}=\frac{1}{49+16}$

$\Rightarrow \quad \frac{v}{65}=\frac{u}{65}=\frac{1}{65}$

$\Rightarrow \quad v=\frac{65}{65}=1$ .... (13)

and $u=\frac{65}{65}=1$ .... (14)

From equation (7) and equation (14), we get

$\frac{1}{x}=1 \Rightarrow x=1$

From equation (8) and equation (13), we get

Hence, the solution of the given pair of the equation is

x = 1, y = 1

Verification, Substituting x = 1, y = 1,

We find that both the equations (1) and (2) are satisfied as shown below:

$\frac{7x-2y}{xy}=\frac{7(1)-2(1)}{(1)(1)}=5$

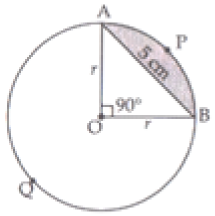
$\frac{8x+7y}{xy}=\frac{8(1)+7(1)}{(1)(1)}=15$

Hence, the solution we have got is correct.

35. Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that θ = 90°

Area of $\triangle$OAB = $\frac{1}{2}$Base $\times$ Altitude = $\frac{1}{2}$r $\times$ r = $\frac{1}{2}r^2$



Area of minor segment APB

$= \frac{\pi r^2 \theta}{360^\circ}$ – Area of $\triangle$AOB

$= \frac{\pi r^2 90^\circ}{360^\circ} - \frac{1}{2} r^2$

$\Rightarrow$ Area of minor segment = $\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ ...(i)

Area of major segment AQB = Area of circle – Area of minor segment

= $\pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$

$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ ...(ii)

Difference between areas of major and minor segment

$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$

$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right $\triangle$OAB,

$r^2 + r^2 = AB^2$

$\Rightarrow 2r^2 = 5^2$

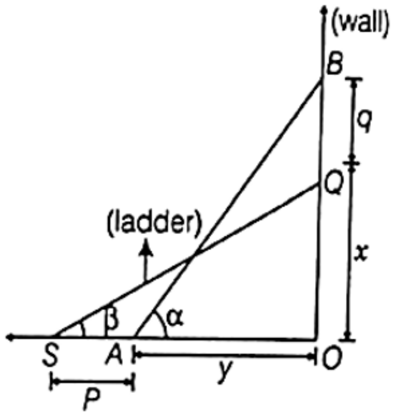
$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right]$cm^2

36. Let QO = x and given that BQ = q, such that BO = q + x is the height of the wall.

Let A O=y and given that SA = q.

$\angle$BAO = α and $\angle QSO = \beta$



In ΔBAO

$\sin\alpha = \frac{BO}{AB}$

[$\because$ by property of sine, $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$]

$\Rightarrow BO = AB\sin\alpha$ ...eq.1

$\cos\alpha = \frac{AO}{AB}$

[$\because$ by property of cosine, $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$]

$\Rightarrow AO = AB\cos\alpha$ eq.2

In ΔQSO

$\sin\beta = \frac{QO}{SQ}$

[$\because$ by property of sine, $\cos\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$]

$\Rightarrow QO = SQ\sin\beta$....eq.3

$\cos\beta = \frac{\text{so}}{\text{sQ}}$

[$\because$ by property of cosine, $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$]

$\Rightarrow SO = SQ\cos\beta$ ...eq.4

Subtracting eq. 2 from eq. 4, we get

$SO - AO = SQ\cos\beta - AB\cos\alpha$

$\Rightarrow SA = SQ\cos\beta - AB\cos\alpha$

[from the above figure, SO - AO = SA = p]

$\Rightarrow p = AB\cos\beta - AB\cos\alpha$

[∵ SQ=AB=length of the ladder]

$\Rightarrow p = AB(\cos\beta - \cos\alpha)$...eq.5

And subtracting eq. 3 from eq. 1, we get

$\mathrm{BO} - \mathrm{QO} = \mathrm{AB}\sin\alpha - \mathrm{SQ}\sin\beta$

$\Rightarrow \mathrm{BQ} = \mathrm{AB}\sin\alpha - \mathrm{SQ}\sin\beta$

[from the above figure, BO - QO = BQ = q]

$\Rightarrow q = AB\sin\alpha - AB\sin\beta$

[∵ SQ = AB = length of the ladder

$\Rightarrow q = AB(\sin\alpha - \sin\beta)$....eq. 6

Dividing eq. 5 and eq. 6, we get

$\Rightarrow \frac{\mathrm{p}}{\mathrm{q}} = \frac{\mathrm{AB}(\cos\beta - \cos\alpha)}{\mathrm{AB}(\sin\alpha - \sin\beta)}$

$\Rightarrow \frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$

Hence proved.