Solution

Class 12 - Mathematics

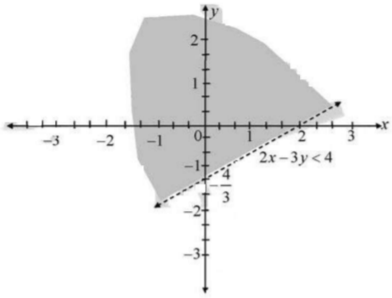
2020-2021 - Paper-7

Section A

1. $3x+4=0 \Rightarrow -x=\frac{4}{3}$
   Direction ratios of the normal to this plane are -1, 0, 0 and $\sqrt{(-1)^2+0^2+0^2}=1$
   Hence the required direction cosines are -1. 0, 0
2. Let $\overrightarrow{a}$ be a vector parallel to the vector having direction ratios 4, - 3, 5. Then, $\vec{a}=4\hat{i}-3\hat{j}+5\hat{k}$. Further, let $\overrightarrow{b}$ be a vector parallel to the vector having direction ratios 3, 4, 5.
   Then,
   $\vec{b}=3\hat{i}+4\hat{j}+5\hat{k}$
   Let θ be the angle between the given vectors. Then.
   $\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{12-12+25}{\sqrt{16+9+25}\sqrt{9+16+25}}=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$
   Thus, the measure of the angle between the given vectors is 60°.
3. Here,it is given the equition of plane is z = 3
   Here direction ratios of normal to the plane is 0,0,1
   Now $\sqrt{0^2+0^2+1^2}$ = 1
   $\therefore$ Direction cosines are 0,0,1 And distance from the origin P = 3.
4. The equation of a plane with intercepts a, b, c on x, y, and z is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$....(i)
   Given, The plane is parallel to ZOX plane
   $\therefore$ Intercept on x- axis =0 $\Rightarrow$ a=0
   and Intercept on z axis = 0 $\Rightarrow$ c=0
   Given, Intercept on y axis =3 $\Rightarrow$ b=3
   Using values of a, b and c in Eq. (i), we get;
   $\frac{y}{3}=1\Rightarrow y=3$
5. We have to find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$.Let $\vec{n}=2\hat{i}-3\hat{j}+6\hat{k}$ and d = 5
   We know that, equation of a plane having distance d from origin and normal vector $\hat{n}$ is
   $\vec{r}.\hat{n}=d$
   $\therefore \quad \vec{r}.\frac{(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+(-3)^2+(6)^2}}=5$
   $\Rightarrow \quad \vec{r}.(2\hat{i}-3\hat{j}+6\hat{k})=5\sqrt{49}$
   $\Rightarrow \quad \vec{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})=35$
6. Let Q divides PR in the ratio λ : 1.
   Then, coordinates of Q are
   $\left(\frac{9\lambda+3}{\lambda+1},\frac{8\lambda+2}{\lambda+1},\frac{-10\lambda-4}{\lambda+1}\right)$
   But, coordinates of Q are given as (5, 4, -6).
   $\therefore \frac{9\lambda+3}{\lambda+1}=5,\frac{8\lambda+2}{\lambda+1}=4,\frac{-10\lambda-4}{\lambda+1}$ = - 6
   All these equations give the same value of λ is equal to $\frac{1}{2}$.
   So, Q divides PR in the ratio $\frac{1}{2}$: 1 or, 1 : 2.
7. Here,we have the direction ratios of $\vec{r}$ are 6, 2, -3.
   And, $\sqrt{6^2+2^2+(-3)^2}=\sqrt{36+4+9}=\sqrt{49}$ =7.
   Hence, the direction cosines of $\vec{r}$ are $\frac{6}{7},\frac{2}{7},\frac{-3}{7}$.
8. We have,max number of passengers = 250
   Profit = 1,500 on an Executive class ticket.

= 1,000 on an economy class ticket.
Seats for executive class $\geq$ 25.
Number of passengers on Executive class = x
number of passengers on economy class = y
Therefore given,
x $\geq$ 25
y $\geq$ 3x
Given x + y = 250
x = 250 - y
x $\leq$ 250 - 3x
4x $\leq$ 250
x $\leq$ 62.5 or 62
x $\leq$ 62.
y $\geq$ 188
Profit = P = x$\times$1500 + 1000$\times$y
Maximize P.
P = 1500x + 1000y
= 1500x + 1000 (250 - x)
= 500x + 250,000
Max profit is when: x = 62
Therefore,maximum profit = 500$\times$62 + 250,000
= 281,000

9.
Here,it is given that 2x - 3y < 4
Taking line 2x - 3y = 4
Taking line 2x - 3y = 4 and substituting
x = 0, y = $= -\frac{4}{3}$
put y = 0, x = 2
Now, we draw the graph of line 2x - 3y = 4
Put (0,0) in 2x - 3y < 4 we get 0 < 4, which is true.
Therefore, shaded region above the line 2x - 3y = 4



10. By addition theorm of probability,we have,
P(A $\cup$ B) = P(A) + P(B) - P(A $\cap$ B)
$= \left(\frac{1}{3}+\frac{1}{4}-\frac{1}{5}\right) = \frac{(20+15-12)}{60} = \frac{23}{60}.$
11. Given: P(A) = 0.3 and P(B) = 0.4
As we know, P (A $\cup$ B) = P(A) + P(B) - P (A $\cap$ B)
Since events are independent so P (A $\cap$ B) = 0. Therefore,
P (A $\cup$ B) = 0.3 + 0.4
⇒ P (A $\cup$ B) = 0.7
12. $\mathrm{P(B|A)} = \frac{\mathrm{P(A}\cap B)}{\mathrm{P(A)}}$
= $\frac{\frac{1}{4}}{\frac{1}{2}}$
$= \frac{1}{2}$

13. We have, n (A) = 3, n(B) = 2

Clearly,

A ∩ B = {3} and n(A ∩ B) = 1

$P(A/B)=\frac{n(A\cap B)}{n(B)}=\frac{1}{2}$ and, $P(B/A)=\frac{n(A\cap B)}{n(A)}=\frac{1}{3}$

14. Total number of outcomes=

P (4 on first die) = $P(A)=\frac{6}{36}=\frac{1}{6}$

P (5 on second die) = $P(B)=\frac{6}{36}=\frac{1}{6}$

$P(A\cap B)=\frac{1}{36}$

$P(A\cap B)=P(A)P(B)$

Therefore , A and B are independent events .

15. A and B be the events such that P(A) $=\frac{5}{11}, P(B)=\frac{6}{11}$ and

$\mathrm{P}(\mathrm{A}\cap\mathrm{B})$

$=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A}\cup\mathrm{B})$

$=\frac{5}{11}+\frac{6}{11}-\frac{7}{11}=\frac{4}{11}$

$\mathrm{P}(\mathrm{A}/\mathrm{B})=\frac{P(A\cap B)}{P(B)}$

$=\frac{4}{11}\div\frac{6}{11}=\frac{4}{6}=\frac{2}{3}$

16. Let E_1 and E_2 be mutually exclusive. Then, $E_1 \cap E_2 = \phi$

$\therefore$ $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$\Rightarrow$ 0.4 = 0.3 + x

$\Rightarrow$ x = 0.1

Section B

17. i. 0.5x+y ≤ 10
    ii. x + y ≤ 15
    iii. 10 and 5
    iv. 175
    v. 250

18. We have, A_1: A_2: A_3 = 4: 4: 2

$P(A_1)=\frac{4}{10}, P(A_2)=\frac{4}{10}$ and $P(A_3)=\frac{2}{10}$

Where A_1, A_2 and A_3 denote the three types of flower seeds.

Let E be the event that seed germinates and $\overline{E}$ be the event that a seed does not germinate.

$\therefore P\left(\frac{E}{A_1}\right)=\frac{45}{100}, P\left(\frac{E}{A_2}\right)=\frac{60}{100}$ and $P\left(\frac{E}{A_3}\right)=\frac{35}{100}$ And $P\left(\frac{\bar{E}}{A_1}\right)=\frac{55}{100}, P\left(\frac{\bar{E}}{A_2}\right)=\frac{40}{100}$ and $P\left(\frac{\bar{E}}{A_3}\right)=\frac{65}{100}$

i. (c)

$\therefore P(E)=P(A_1)\cdot P\left(\frac{E}{A_1}\right)+P(A_2)\cdot P\left(\frac{E}{A_2}\right)+P(A_3)\cdot P\left(\frac{E}{A_3}\right)$

$=\frac{4}{10}\cdot\frac{45}{100}+\frac{4}{10}\cdot\frac{60}{100}+\frac{2}{10}\cdot\frac{35}{100}$

$=\frac{180}{1000}+\frac{240}{1000}+\frac{70}{1000}=\frac{490}{1000}=0.49$

ii. (b)

$P\left(\frac{\bar{E}}{A_3}\right)=1-P\left(\frac{E}{A_3}\right)=1-\frac{35}{100}=\frac{65}{100}$ [as given above]

iii. (d)

$$P\left(\frac{A_2}{\bar{E}}\right)=\frac{P(A_2)\cdot P\left(\frac{\bar{E}}{A_2}\right)}{P(A_1)\cdot P\left(\frac{\bar{E}}{A_1}\right)+P(A_2)+P\left(\frac{\bar{E}}{A_2}\right)+P(A_3)\cdot P\left(\frac{\bar{E}}{A_3}\right)}$$

$$=\frac{\frac{4}{10}\cdot\frac{40}{100}}{\frac{4}{10}\cdot\frac{55}{100}+\frac{4}{10}\cdot\frac{40}{100}+\frac{2}{10}\cdot\frac{65}{100}}=\frac{\frac{160}{1000}}{\frac{220}{1000}+\frac{160}{1000}+\frac{130}{1000}}$$

$$=\frac{\frac{160}{1000}}{\frac{510}{1000}}=\frac{16}{51}$$

iv. (b)

$$P\left(\frac{A_2}{\bar{E}}\right)=\frac{P(A_1)\cdot P\left(\frac{\bar{E}}{A_1}\right)}{P(A_1)\cdot P\left(\frac{\bar{E}}{A_1}\right)+P(A_2)+P\left(\frac{\bar{E}}{A_2}\right)+P(A_3)\cdot P\left(\frac{\bar{E}}{A_3}\right)}$$

$$=\frac{\frac{4}{10}\cdot\frac{55}{100}}{\frac{4}{10}\cdot\frac{55}{100}+\frac{4}{10}\cdot\frac{40}{100}+\frac{2}{10}\cdot\frac{65}{100}}=\frac{\frac{220}{1000}}{\frac{220}{1000}+\frac{160}{1000}+\frac{130}{1000}}$$

$$=\frac{\frac{220}{1000}}{\frac{510}{1000}}=\frac{22}{51}$$

v. (a)

$$P\left(\frac{\bar{E}}{A_1}\right)=1-P\left(\frac{E}{A_1}\right)=1-\frac{45}{100}=\frac{55}{100}$$

Section C

19. The equation of the plane with intercepts a,b,c on X-axis, Y-axis and z-axis respectively is,

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Here, a = 2, b = 3, c = 4.

Therefore, required equation is,

$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$

$\Rightarrow 6x+4y+3z=12$

20. Equation of plane through three points (2, 2, 1), (3, 0, 1) and (4, -1, 0) is

$[(\vec{r}-(2\hat{i}+2\hat{j}+\hat{k})]\cdot[(\hat{i}-2\hat{j})\times(\hat{i}-\hat{j}-\hat{k})]=0$

i.e. $\vec{r}\cdot(2\hat{i}+\hat{j}+\hat{k})=7$ or 2x + y + z - 7 = 0 ...(i)

Equation of line through (3, -4, -5) and (2, -3, 1) is

$\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}$ ....(ii)

Any point on line (ii) is $(-\lambda+3,\lambda-4,6\lambda-5)$.

This point lies on plane (i).

Therefore, $2(-\lambda+3)+(\lambda-4)+(6\lambda-5)-7=0$,

i.e., $\lambda=2$

Hence the required point is (1, -2, 7).

21. We have,

$r\cdot(i+j+\hat{k})+17=0$

$\Rightarrow(x\hat{i}+y\hat{j}+z\hat{k})(i+\hat{j}+\hat{k})+17=0$

$\Rightarrow$ x + y + z + 17 = 0 $\because P=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

$\Rightarrow P=\left|\frac{1(1)+1(2)+1(5)+17}{\sqrt{(1)^2+(1)^2+(1)^2}}\right|\Rightarrow P=\left|\frac{1+2+5+17}{\sqrt{3}}\right|\Rightarrow P=\frac{25}{\sqrt{3}}$ units .

OR

The equation of any plane passing through (2, 2, 1) is

a(x-2) + b(y-2) + c(z-1) = 0 ....(1)

It is also given that (1) is passing through (9,3,6) . Therefore,

a(9-2) + b(3-2) + c(6-1) = 0 ...(2)

It is also given that (1) is perpendicular to the plane 2x + 6y + 6z = 1 . So,

2a + 6b + 6c = 0

$\Rightarrow$ a + 3b + 3c = 0 ....(3)

Solving (1), (2) and (3), we obtain

$$\begin{vmatrix}x-2 & y-2 & z-1\\ 7 & 1 & 5\\ 1 & 3 & 3\end{vmatrix}=0$$

$\Rightarrow$ -12(x-2) - 16(y-2) + 20(z-1) = 0

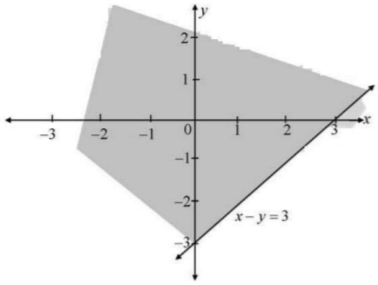
$\Rightarrow$ 3(x-2) + 4(y-2) - 5(z-1) = 0

$\Rightarrow$ 3x + 4y - 5z = 9

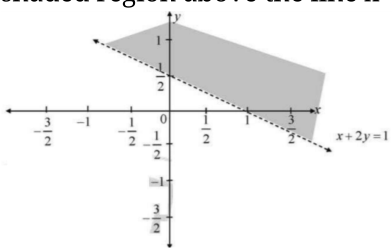
Hence the required equation of plane.

22. Let firm produces x units of product A units product A and Y units product B
Then total profit = Rs. (2x + 2y)
Total time available for M_1 to produce both E = x + y $\leq$ 400
Total time available for M_2 to produce both A and B = 2x + y $\leq$ 600
Therefore, mathematically, L.P.P can be given as:
Maximize $z = 2x + 2y$ subject to constraints
$x + y \leq 400$ [first constraint]
$2x + y \leq 600$ [second Constraint]
$x \geq 0, y \geq 0$ non-negativity constraints

23. Here, it is given that x - y < 3
Taking line x - y = 3
taking line 3x - 2y = 12 and substituting x = 0 y = -3
Putting y = 0, x = 3
Now, we draw the graph of line x - y = 3
putting, (0, 0) in x - y < 3 we get 0 - 0 < 3, which is true.
Therefore, shaded region towards origin including line x - y =3



24. Here it is given that, x + 2y > 1
Taking line x + 2y = 1
Taking line x + 2y = 1 and
Substituting x = 0, $y = \frac{1}{2}$
Putting y = 0, x = 1
Now we draw the graph of line x + 2y = 1
Put, (0,0) in x +2y > 1 we get 0 > 1 which is false
shaded region above the line x + 2y = 1



25. Let A denote the event that first engine is available when needed and B, the event that second engine is available when needed. Then, P (A) = P (B) = 0.95.
Required probability = P (A) + P (B) - 2 P (A $\cap$ B)
= P (A) + P (B) - 2 P (A) $\times$ P (B) [as A and B are independent]
= 0.95 + 0.95 - 2 $\times$ 0.95 $\times$ 0.95 = 0.095

26. The sample space of the given experiment will be:
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
Here, E: at most two tails
And F: at least one tail
⇒ E = {HHH, HHT, HTH, THH, HTT, THT, TTH}
And F = {HHT, HTH, THH, HTT, THT, TTH, TTT}
$P(E \cap F) = \{\text{HHT, HTH, THH, HTT, THT, TTH}\}$
So, P(E) = $\frac{7}{8}$, P(F) = $\frac{7}{8}$, $P(E \cap F) = \frac{6}{8} = \frac{3}{4}$

By definition of conditional probability, $P(E|F) = \frac{P(E \cap F)}{P(F)}$

$\Rightarrow P(E|F) = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{6}{7}$

27. Given;

A= {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

So, n(A)= 6, n(S)= $(6)^2$= 36

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

And B = {(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)}

$\Rightarrow$ n(B)= 6

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

A∩B = [(5,5), (6,6)]

$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$P(A) \cdot P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Thus,

P (A ∩ B) $\neq$ P(A). P(B)

So, A and B are not independent events.

28. A is an event of getting 5 on the first throw and 6 on the second throw.

A = {(5,6,1), (5,6,2), (5,6,3), (5,6,4), (5,6,5), (5,6,6)}

Also, B is an event of getting 3 or 4 on the third throw.

$\therefore A \cap B = \{(5,6,3), (5,6,4)\}$

Required probability, $P(A|B) = \frac{n(A \cap B)}{n(A)} = \frac{2}{6} = \frac{1}{3}$

OR

Let X be a random variable denoting the number of success in 8 throws.

Here, n=8

p = probability of getting 5 or 6

$= \frac{2}{6} = \frac{1}{3}$ and $q = \frac{2}{3}$

Mean (np) $= \frac{8}{3}$

Variance (npq) $= \frac{16}{9}$

Standard deviation $= \sqrt{\text{Variance}} = \frac{4}{3}$

So, mean = 2.66 and standard deviation = 1.33

Section D

29. Equation of any plane through the intersection of given planes can be taken as

$(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$ ...(1)

Since the point (2, 2, 1) lies in this plane,therefore,we get,

(3(2) - 2 + 2(1) - 4) + $\lambda[2 + 2 + 1 - 2] = 0$

$\Rightarrow$2 + λ(3) = 0

$\Rightarrow \lambda = -\frac{2}{3}$. Put value of λ in eq (i), we get,

$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$

7x - 5y + 4z - 9 = 0

30. We know that,

equation of the plane passing through (x_1, y_1, z_1) having Direction ratios a, b,c is; a

$(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Equation of plane passing through the point A (1, 2, 1) is given as

$a(x - 1) + b(y - 2) + c(z - 1) = 0$ .....(i)

Now, DR's of line PQ where $P(1,4,2)\ and\ Q(2,3,5)\ are\ (2 - 1, 3 - 4, 5 - 2) = (1, -1, 3)$.

Plane (i) is perpendicular to line PQ.

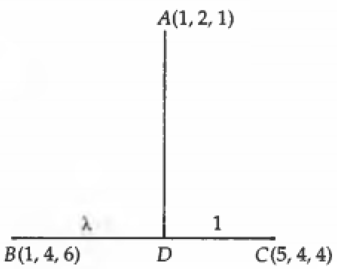
$\therefore$ Direction ratios of plane (i) are (1, -1, 3) [$\because$ Direction ratios normal to the plane are proportional]

i.e. $a = 1, b = -1, c = 3$

On putting values of a, b and c in Eq. (i), we get the required equation of plane as

$1(x-1)-1(y-2)+3(z-1)=0$
$x-1-y+2+3z-3=0$
$x-y+3z-2=0$ ......(ii)
Now, the given equation of line is
$\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$....(iii)
Direction ratios of this line are $(2,-1,-1)$ and passing through the point $(-3,5,7)$.
Direction ratios of normal to the plane (ii) are $(1,-1,3)$.
To check whether the line is perpendicular to the plane, we use the condition $a_1a_2+b_1b_2+c_1c_2=0$
$2(1)-1(-1)-1(3)=2+1-3=0$
So, line (iii) is perpendicular to plane (i).
Therefore, distance of the point $(-3,5,7)$ from the plane (ii)
$\Rightarrow d=\left|\frac{(-3)(1)+(5)(-1)+7(3)-2}{\sqrt{(1)^2+(-1)^2+(3)^2}}\right|$
$=\left|\frac{-3-5+21-2}{\sqrt{1+1+9}}\right|$
$=\left|\frac{11}{\sqrt{11}}\right|=\left|\frac{(\sqrt{11})^2}{\sqrt{11}}\right|$
$=\sqrt{11} units$

31. Let D be the foot of the perpendicular drawn from A on BC, and let D divide BC in the ratio λ : 1. Then, coordinates of D are
$\left(\frac{5\lambda+1}{\lambda+1},\frac{4\lambda+4}{\lambda+1},\frac{4\lambda+6}{\lambda+1}\right)$ ....(1)
Now, $\overrightarrow{AD}$ = Position vector of D - Position vector of A
$\Rightarrow \overrightarrow{AD}=\left(\frac{5\lambda+1}{\lambda+1}-1\right)\hat{i}+\left(\frac{4\lambda+4}{\lambda+1}-2\right)\hat{j}+\left(\frac{4\lambda+6}{\lambda+1}-1\right)\hat{k}$
$\Rightarrow \overrightarrow{AD}=\left(\frac{4\lambda}{\lambda+1}\right)\hat{i}+\left(\frac{2\lambda+2}{\lambda+1}\right)\hat{j}+\left(\frac{3\lambda+5}{\lambda+1}\right)\hat{k}$
also$\overrightarrow{BC}$ = Position vector of C - Position vector of B
$\Rightarrow \overrightarrow{BC}$ = $(5\hat{i}+4\hat{j}+4\hat{k})-(\hat{i}+4\hat{j}+6\hat{k})=4\hat{i}+0\hat{j}-2\hat{k}$.
It is given that AD is perpendicular to BC.
$\therefore \overrightarrow{AD}\perp\overrightarrow{BC}$
$\Rightarrow \overrightarrow{AD}\cdot\overrightarrow{BC}=0$
$\Rightarrow \left\{\left(\frac{4\lambda}{\lambda+1}\right)\hat{i}+\left(\frac{2\lambda+2}{\lambda+1}\right)\hat{j}+\left(\frac{3\lambda+5}{\lambda+1}\right)\hat{k}\right\}\cdot(4\hat{i}+0\hat{j}-2\hat{k})$ = 0
$\Rightarrow 4\left(\frac{4\lambda}{\lambda+1}\right)+0\left(\frac{2\lambda+2}{\lambda+1}\right)-2\left(\frac{3\lambda+5}{\lambda+1}\right)$ = 0
$\Rightarrow \frac{16\lambda}{\lambda+1}+0-2\frac{(3\lambda+5)}{\lambda+1}=0$



$\Rightarrow$ 16λ - 6λ - 10 = 0
$\Rightarrow$ λ = 1
Substituting λ = 1 in (i) we obtain,
Coordinates of D are (3, 4, 5).

OR

Given lines can be written as

$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$

Now, on comparing both lines with

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{b}$, we get ,

x_1 = 5, y_1 =7, z_1 = -3, a_1 = 4, b_1 = 4,c_1 = -5 and x_2 = 8, y_2 = 4, z_2 = 5, a_2 = 7, b_2 = 1 , c_2 = 3

We know that if given lines are coplanar, then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\text{LHS}=\begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}=\begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

= 3(12+5) + 3(12+35) + 8(4+28)

=3×17 + 3×47 + 8(-24)

=21 + 141 - 192 = 192 - 192 = 0 = RHS

Therefore, given lines are coplanar.

32. If h and t represents heads and tails respectively, then when a coin is tossed three times, the sample space becomes

S = {(HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)},n(S)=8

Let E be the event of head occurring on the third toss, then the number of cases favorable to the event,

E = {(HHH, HTH, THH, TTH)}, n(E) = 4

Therefore,we have,

$P(E)=\frac{n(E)}{n(S)}=\frac{4}{8}=\frac{1}{2}$

Let F be the event that head occurs on first two toss, then the favorable outcomes will be

F = {(HHH,HHT)}, n(F) = 2

So the corresponding probability will be

$P(F)=\frac{n(F)}{n(S)}=\frac{2}{8}=\frac{1}{4}$

And the favorable outcome for getting head on all the three toss will be

(E∩F)={(HHH)}, n(E∩F)=1

And the corresponding probability becomes

$P(E\cap F)=\frac{n(E\cap F)}{n(F)}=\frac{1}{8}$

So if head occurs on first two tosses, the probability of getting head on third tosses

$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{8}}{\frac{1}{4}}=\frac{1}{2}$

33. Let E_1, E_2 and A be the events defined as follows:

E_1 = Selecting a bag from the first group, E_2 = Selecting a bag from the second group and, A = Ball drawn is white

Since there are 5 bags out of which 3 bags belong to first group and 2 bags to second group.Therefore,we have,

$P(E_1)=\frac{3}{5}, P(E_2)=\frac{2}{5}$

If E_1 has already occurred, then a bag from the first group is chosen. The bag chosen contains 5 white balls and 3 black balls. Therefore, the probability of drawing a white ball from it is 5/8.

∴ P(A/E_1) = 5/8

Similarly, P (A/E_2) = 2/6 = 1/3.

We have to find P (E_1/M), i.e. given that the ball drawn is white, what is the probability that it is drawn from a bag of the first group.

By Baye's theorem, required probability is given by,

$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}=\frac{\frac{3}{5}\times\frac{5}{8}}{\frac{3}{5}\times\frac{5}{8}+\frac{2}{5}\times\frac{1}{3}}=\frac{45}{61}$

34. A white ball and a red ball can be drawn in three mutually exclusive ways:
(I) Selecting bag I and then drawing a white and a red ball from it
(II) Selecting bag II and then drawing a white and a red ball from it
(II) Selecting bag III and then drawing a white and a red ball from it
Consider the following events:
E_1 = Selecting bag I
E_2 = Selecting bag II
E_3 = Selecting bag II
A = Drawing a white and a red ball
It is given that one of the bags is selected randomly.
$\therefore P(E_1) = \frac{1}{3}$
$P(E_2) = \frac{1}{3}$
$P(E_3) = \frac{1}{3}$
Now,
$P(A/E_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{3}{15}$
$P(A/E_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2}{6}$
$P(A/E_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{12}{66}$
Using the law of total probability, we get
Required probability = P(A) = $P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$
$= \frac{1}{3} \times \frac{3}{15} + \frac{1}{3} \times \frac{2}{6} + \frac{1}{3} \times \frac{12}{66}$
$= \frac{1}{15} + \frac{1}{9} + \frac{2}{33}$
$= \frac{33+55+30}{495} = \frac{118}{495}$

OR

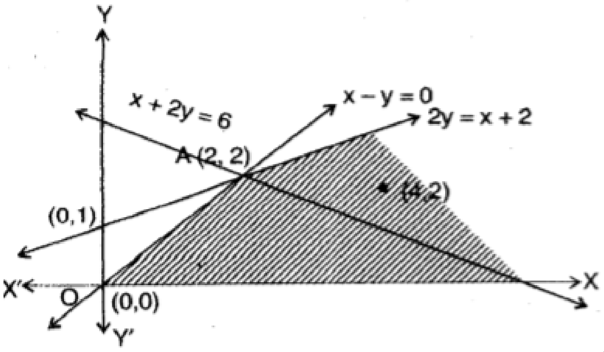
As it is clear from the given graph, the coordinates of the corner points O, A, E and Dare given as (0, 0), (52, 0), (144, 16) and (0, 38), respectively. Also given region is bounded.
Given that the objective function, Z = 3x + 4y to be maximised.
∵ Also converting the given inequalities into their equations to find their points of intersection ,
2x + y = 104............(i) and
2x + 4y = 152............(ii)
solving above equations (i) and (ii), we get
⇒ -3y = -48
⇒ y = 16 and x = 44.
Hence the point of intersection of the lines (i) and (ii) is (16,44)
The table below gives the values of the objective function Z, at the corner points of the feasible region.

Corner Points	**Corresponding value of Z**
(0, 0)	Z = 0
(52, 0)	Z = 156
(44, 16)	Z = 196 (maximum)
(0, 38)	Z = 152

Hence, Z attains its maximum at the point (44,16) and its maximum value is 196.

35. According to the question,
Maximize $Z = x + 2y$
Subject to the constraints
$x - y \geq 0$
$2y \leq x + 2$
such that $x \geq 0, y \geq 0$
We draw line $x - y = 0$ and $2y = x + 2$ and shaded the feasible region with respect to constraints sign.
We observe that the feasible region is unbounded and corner points are $O(0,0), A(2,2)$ only. Evaluating Z at

all corner points,



Corner Points	**Z = x + 2y**
$(0,0)$	$Z=0\ (Smallest)$
$(2,2)$	$Z=6\ (largest)$

We are to determine the maximum value. As the feasible region is unbounded we can not say whether the largest value Z = 6 is maximum or not.

We draw the half plane $x+2y>6$ and notice that it has common points with feasible region. There does not exist any maximum value. For testing we let $x=4, y=2$, this point lie in the feasible region and at $(4,2)$ the value of $Z=8$ i.e., greater than '6'.

Section E

36. Given equations of planes are

2x + y - z - 3 = 0 ....(i)

and 5x - 3y + 4z + 9 = 0 ....(ii)

Let the required equation of plane which passes through the line of intersection of planes (i) and (ii) be

$(2x+y-z-3)+\lambda(5x-3y+4z+9)=0$ ....(iii)

$\Rightarrow x(2+5\lambda)+y(1-3\lambda)+z(-1+4\lambda)+(-3+9\lambda)=0$ ......(iv)

Here, Direction Ratio's of the plane are

$(2+5\lambda, 1-3\lambda, -1+4\lambda)$

Also, given that the plane (iv) is parallel to the line, whose equation is

$\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$

Direction ratio's of the line are (2, 4, 5).

Since the plane is parallel to the line.

Hence, normal to the plane is perpendicular to the line,

i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, $a_1=2+5\lambda, b_1=1-3\lambda, c_1=-1+4\lambda$

and $a_2 = 2$, $b_2 = 4$, $c_2 = 5$

$\therefore \quad (2+5\lambda)2+(1-3\lambda)4+(-1+4\lambda)5=0$

$\Rightarrow \quad 4+10\lambda+4-12\lambda-5+20\lambda=0$

$\Rightarrow \quad 18\lambda+3=0 \Rightarrow \lambda=-\frac{3}{18}=-\frac{1}{6}$

On putting $\lambda=-\frac{1}{6}$in Eq. (iii), we get the required equation of plane as

$(2x+y-z-3)-\frac{1}{6}(5x-3y+4z+9)=0$

12 + 6y - 6z - 1 8 - 5x + 3y - 4z- 9 = 0

7x + 9y - 10z - 27 = 0

OR

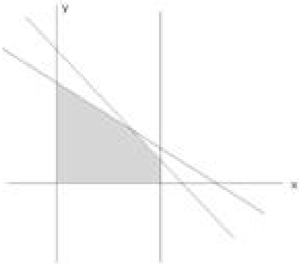
The equation of the plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Distance of this plane from the origin is given to be p.

$\therefore p=\frac{\left|\frac{1}{a}\times 0+\frac{1}{b}\times 0+\frac{1}{c}\times 0-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$

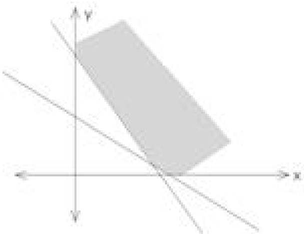
$\Rightarrow p=\frac{1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$

$\Rightarrow \frac{1}{p}=\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$
$\Rightarrow \frac{1}{p^2}=\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2$
$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

37. 

Let x toys of type A and y toys of type B
$Z=\frac{15}{2}x+5y$
$12x+6y\leqslant 360$
$\Rightarrow 2x+y\leqslant 60$
$18x\leqslant 360$
$\Rightarrow x\leqslant 20$
$6x+9y\leqslant 360$
$\Rightarrow 2x+3y\leqslant 120$
$x\geqslant 0, y\geqslant 0$
Solving these equations, we get,
x=15 ,y=30
maximum profit=Rs [$\frac{15}{2}$(15)+5(30)]= Rs 262.5.

OR



Let the farmer use x Kg of F_1 and y Kg of F_2.
Z = 6x + 5y
$\frac{10x}{100}+\frac{5y}{100}\geqslant 14$
$\frac{6x}{100}+\frac{10y}{100}\geqslant 14$
$x\geqslant 0, y\geqslant 0$
On solving these equations we get x = 100 and y = 80
Minimum cost
$z=6\times 100+5\times 80=600+400=1000$
Thus, minimum cost = Rs 1000.
Hence, 100 kg of fertilizer F_1 and 80 kg of fertilizer F_2 should be used so that nutrient requirements are met at minimum cost of Rs 1000.

38. Let E_1 = Event that 1 occurs is a die
E_2 = Event that 1 does not occur in a die
A = Event that the man reports that 1 occur in a die
Then P(E_1)=$\frac{1}{6}$and P(E_2)= $\frac{5}{6}$
$\therefore$ P (man reports that 1 occurs when 1 occur)
$=P\left(\frac{A}{E_1}\right)=\frac{3}{5}$
Thus, by Baye's theorem, we get
P(get actually 1 when he reports that 1 occur)

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{1}{6}\times\frac{3}{5}}{\frac{1}{6}\times\frac{3}{5}+\frac{5}{6}\times\frac{2}{5}}=\frac{3}{13}$$

OR

Let E denote the event that the person selected is actually having HIV and A the event that the person's HIV test is diagnosed as + ve.

Let E' denote the event that person selected is actually not having HIV.

Clearly, {E, E'} is a partition of the sample space of all people in the population.

We are given that

$$P(E)=0.1\%=\frac{0.1}{100}=0.001$$

$$P(E')=1-P(E)=0.999$$

$$P(A/E)=90\%=\frac{90}{100}=0.9$$

$$P(A/E')=1\%=\frac{1}{100}=0.01$$

By Bayes theorm,we have,

$$P(E/A)=\frac{P(E)P(A/E)}{P(E)P(A/E)+P(E')P(A/E')}$$

$$=\frac{0.001\times0.9}{0.001\times0.9+0.999\times0.01}$$

$$=\frac{9}{9+99.9}$$

$$=\frac{90}{1089}$$

= 0.083