## Solution

## Class 12 - Mathematics

## 2020-21 paper 8

### Part - A Section - I

1. Suppose f is not one-one.
   Then there exists two elements, say 1 and 2 in the domain whose image in the co-domain is the same.
   Also, the image of 3 under f can be only one element.
   Therefore, the range set can have at the most two elements of the co-domain {1, 2, 3}, showing that f is not onto, a contradiction. Hence, f must be one-one.

   OR

   The given relation R in the set {1, 2, 3} is given by R = {(1, 2), (2, 1)}
   We know that, for a relation to be transitive,
   (x,y) $\in$ R and (y,z) $\in$ R $\Rightarrow$ (x, z) $\in$ R.
   Here, (1, 2) $\in$ R and (2, 1) $\in$ R
   but (1, 1) $\notin$ R.
   Therefore, R is not transitive.
2. f is a function since each element of A in the first place in the ordered pairs is related to only one element of A in the second place while g is not a function because 1 is related to more than one element of A, namely, 2 and 3.

   OR

   R is reflexive and symmetric, but not transitive since for (1, 0) $\in$ R and (0, 3) $\in$ R whereas (1, 3) $\notin$ R.
3. We have, f(x) = x - [x]=$\{x\}$ ( fractinal part of x)
   **Injection test:**
   f(x) = 0 for all x $\in$ Z
   So, f is a many-one function.
   **Surjection test:**
   Range (f) = [0, 1) $\neq$ R.
   So, f is an into function.
   Therefore, f is neither one-one nor onto.
4. Order of matrix = Number of rows $\times$ Number of columns.
   Thus order $= (1 \times 4)$
5. We have to find,
   A - 2B + 3C i,e,
   A - 2B + 3C = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2\left(\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\right) + 3\left(\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}\right)$
   $= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$
   $= \begin{bmatrix} -6 & 13 \\ 16 & 4 \end{bmatrix}$

   OR

   $$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$
   $$\Rightarrow A' = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$
   $\Rightarrow$A' = -A
   Hence Proved.
6. After finding determinant we will get a trigonometric identity.
   $2\cos^2\theta + 2\sin^2\theta$ = 2

Since we know
$\sin^2\theta+\cos^2\theta=1$

7. We know that $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|+C$
$\int\sqrt{x^2-16}dx=\int\sqrt{x^2-4^2}dx$
$=\frac{x}{2}\cdot\sqrt{x^2-4^2}-\frac{16}{2}\log|x+\sqrt{x^2-16}|+C$
$=\frac{x}{2}\cdot\sqrt{x^2-16}-8\log|x+\sqrt{x^2-16}|+C$ [a = 4]

OR

On dividing ($x^4$ + 1) by ($x^2$ + 1), we get
$\int\left(\frac{x^4+1}{x^2+1}\right)dx=\int\left[x^2-1+\frac{2}{(x^2+1)}\right]dx$
$=\int x^2dx-\int dx+2\int\frac{1}{x^2+1}dx$
$=\frac{x^3}{3}-x+2\tan^{-1}x+C$,where C is constant of integration.

8. we have, $\int_0^a f(x)dx=\frac{a^2}{2}+\frac{a}{2}sin\ a+\frac{\pi}{2}cos\ a$
Differentiating w.r.t a,we get,
f(a)=a+$\frac{1}{2}(sin\ a+acos\ a)-\frac{\pi}{2}sin\ a$
put a=$\frac{\pi}{2}$, $f(\frac{\pi}{2})=\frac{\pi}{2}+\frac{1}{2}-\frac{\pi}{2}=\frac{1}{2}$

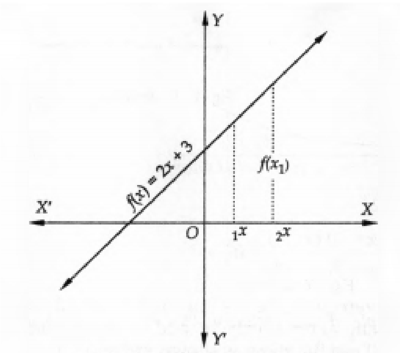
9. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 2
$\therefore$ its order = 2 and degree = 2.

OR

It is given that equation is $\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$
We can see that the highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$
Thus, its order is two. The given differential equation is not a polynomial equation in its derivative.
Therefore, its degree is not defined.

10. Let f(x)= $x^7$ + $14x^5$ + $16x^3$ + 30x - 560 = 0
$f'(x)=7x^6+70x^4+48x^2+30>0,\ \forall\ x\in R$
So, f(x) is increasing.
Hence, f(x)=0 has only one solution.
Or according to the rule of signs, f( x) has the sign change in the coefficients only once ( between the coefficient of x and the constant term) . So there is exactly only one positive real root/solution for f( x).

11. Let $x_1,x_2\in R\ and\ let\ x_1<x_2.Then,$
$x_1<x_2\Rightarrow 2x_1<2x_2\Rightarrow 2x_1+3<2x_2+3\Rightarrow f(x_1)<f(x_2)$
$x_1<x_2\Rightarrow f(x_1)<f(x_2)$ for all $x_1,x_2\in R.$ So, $f(x)$ is strictly increasing function on R This result also follows from the below graph.




12. 5 Seconds is a time period, it has only magnitude i.e; 5 and has no direction, So it is Scalar.

13. We know that the perpendicular distance of a point with position vector $\overrightarrow{r_1}$ from the plane $\vec{r}\vec{n}=q$ is given by $P=\frac{|\vec{r}_1\vec{n}+q|}{|\vec{n}|}$,
Here, $\vec{r}_1=\hat{i}+\hat{j}+2\hat{k}$,$\vec{n}=2\hat{i}-2\hat{j}+4\hat{k}$ and q=5

$\therefore P=\frac{|(i+j+2\hat{k})(2\hat{i}-2\hat{j}+4\hat{k})+5|}{|2\hat{i}-2\hat{j}+4\hat{k}|}=\left|\frac{2-2+8+5}{\sqrt{(2)^2+(-2)^2+(4)^2}}\right|=\frac{13}{2\sqrt{6}}$ units

$=\frac{13\sqrt{6}}{12}$units.

14. We know that the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are perperndicular if $a_1a_2+b_1b_2+c_1c_2=0$.

The given planes are x - 2y + 4z = 10 and 18x + 17y + 4z = 49

$\Rightarrow a_1=1; b_1=-2; c_1=4; a_2=18; b_2=17; c_2=4$

Now, $a_1$ $a_2$ + $b_1$ $b_2$ + $c_1$ $c_2$ = (1)(18) + (-2)(17) + (4)(4) = 18 - 34 + 16 = 0

So, the given planes are perpendicular to each other.

15. Expanding the given determinant along 1st row, we have

$\Delta=3\cdot\begin{vmatrix}2 & -3\\ 1 & 7\end{vmatrix}-4\cdot\begin{vmatrix}-6 & -3\\ 8 & 7\end{vmatrix}+5\cdot\begin{vmatrix}-6 & 2\\ 8 & 1\end{vmatrix}$

=3. (14 + 3) - 4.(-42 + 24) + 5.(-6 - 16)

= 13

16. We know that

P(S|F) = $\frac{P(S\cap F)}{P(F)}=\frac{P(F)}{P(F)}=1$

Also, P(S|F) = $\frac{P(F\cap F)}{P(F)}=\frac{P(F)}{P(F)}=1$

Thus,P(S|F) = P(F|F) = 1

**Section - II**

17. i. (a) ₹ 2
ii. (c) ₹17
iii. (a) ₹7
iv. (a) ₹20
v. (a) ₹22

18. i. (a) Team B
ii. (b) 1.4 KN
iii. (c) 2.4 radian
iv. (a) $2\sqrt{5}$ KN
v. (b) 4 KN

**Part - B Section - III**

19. Let $\sin^{-1}\left(\frac{-1}{2}\right)=y$

$\Rightarrow \sin y=-\frac{1}{2}$

$\Rightarrow \sin y=-\sin\frac{\pi}{6}$

$\Rightarrow \sin y=\sin\left(-\frac{\pi}{6}\right)$

Since, the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Therefore, principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

20. Let $A=\begin{bmatrix}5 & 4\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}1 & -2\\ 1 & 3\end{bmatrix}$

$\Rightarrow$ AX = B

Or, X = $A^{-1}$B

|A| = 1

Now Cofactors of A are

$C_{11}$ = 1 $C_{12}$ = – 1

$C_{21}$ = – 4 $C_{22}$ = 5

$\Rightarrow$ adj A = $\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^T$

= $\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$ adj $A$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

So, X = $\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

Hence, X = $\begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$

OR

We have given that,

ar( $\Delta$ABC) = 3 sq units .

$$\Leftrightarrow \frac{1}{2}\cdot\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Leftrightarrow \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 6$$

$$\Leftrightarrow (-1)\cdot\begin{vmatrix} 1 & 3 \\ k & 0 \end{vmatrix} = \pm 6 \Leftrightarrow 3k = \pm 6 \Leftrightarrow k = \pm 2$$

Hence, k = ±2

21. Given: $f(x) = |\cos x|$ ....(i)

f(x) has a real and finite value for all $x \in R$.

$\therefore$ Domain of f(x) is R.

Let g(x) = cos x and $h(x) = |x|$

Since g(x) and h(x) being cosine function and modulus function are continuous for all real x

Now, $(goh)x = g\{h(x)\} = g(|x|) = \cos|x|$ being the composite function of two continuous functions is continuous, but not equal to f(x)

Again, $(hog)x = h\{g(x)\} = h(\cos x) = |\cos x| = f(x)$[Using eq. (i)]

Therefore, $f(x) = |\cos x| = (hog)x$ being the composite function of two continuous functions is continuous.

22. Given: The equation of the curve is-

$y = 2x^3 - 3$ ... (i)

Differentiating with respect to x, we get

$\frac{dy}{dx} = 6x^2$

Now, $m_1$ = (Slope of the tangent at point x = 2) $= \left(\frac{dy}{dx}\right)_{x=2} = 6 \times (2)^2 = 24$

and $m_2$ = (Slope of the tangent at point x = -2) $= \left(\frac{dy}{dx}\right)_{x=-2} = 6(-2)^2 = 24$

Clearly, $m_1 = m_2$.

Thus, the tangents to the given curve at the points where x = 2 and x = - 2 are parallel.

23. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and cos x is the second function.

Using Integration by part

$$\int \mathrm{a.b}\cdot\mathrm{dx} = \mathrm{a}\int\mathrm{bdx} - \int\left[\frac{\mathrm{da}}{\mathrm{dx}}\cdot\int\mathrm{bdx}\right]\mathrm{dx}$$

$\Rightarrow \int$ x cos x dx = x $\int$ cos x-$\int\left[\frac{dx}{dx}\int\cos x\,dx\right]dx$

= x sin x - $\int$ 1.sin x dx

= x sin x + cos x + c

OR

Let I = $\int e^x (\cos x - \sin x)\, dx$

$= \int e^x \cos x\, dx - \int e^x \sin x\, dx$

Integrating by parts

$= e^x \cos x - \int e^x \left(\frac{d}{dx}\cos x\right) dx - \int e^x \sin x\, dx$

$= e^x \cos x + \int e^x \sin x\, dx - \int e^x \sin x\, dx$

$= e^x \cos x + c$

$\therefore \int e^x (\cos x - \sin x)\, dx = e^x \cos x + c$

24. $y^2 = 4x$

$x = 3$

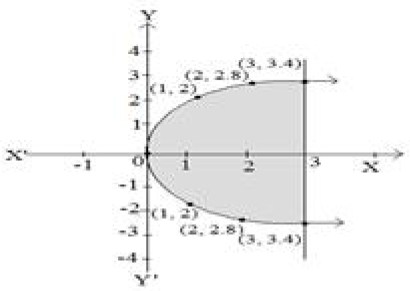
$= 2\int_0^3 \sqrt{4x} dx$

$= 4\int_0^3 \sqrt{x}\, dx$

$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^3$

$= \frac{8}{3}\left[3^{3/2} - 0\right]$

$= 8\sqrt{3}$sq. units




25. Given differential equation is $2x\frac{dy}{dx} + y = 6x^3$

$\Rightarrow \frac{dy}{dx} + \frac{1}{2x} \cdot y = 3x^2$

This is of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{1}{2x}$ and Q = $3x^2$

Thus the differential equation is linear

Now, $IF = e^{\int Pdx}$

$= e^{\int \frac{dx}{2x}} = e^{\frac{1}{2}\log x} = x^{1/2}$

There fore the solution is given by

$y(IF) = \int (IF)Q\, dx + C$

$\Rightarrow y \cdot x^{1/2} = \int x^{1/2} \cdot 3x^2 dx + C$

$\Rightarrow y \cdot x^{1/2} = 3\int x^{3/2} dx + C \Rightarrow y \cdot x^{1/2} = 3 \cdot \frac{x}{7/2} + C$

$\Rightarrow y \cdot x^{1/2} = \frac{6}{7} x^{1/2} + C \Rightarrow y = \frac{6}{7} \cdot x^3 + \frac{C}{\sqrt{x}}$

This is the required solution of given differential equation.

26. $\frac{dx}{dt} = 3a\sin^2 t.\cos t dt$

$\frac{dy}{dt} = -3b\cos^2 t \sin t dt$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3bcos^2\ tsin\ t}{3asin^2\ tcos\ t} = \frac{-b}{a}\cot t$

$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{2}} = \frac{-b}{a} \times cot\ \frac{\pi}{2} = \frac{-b}{a} \times 0 = 0$

When $t = \frac{\pi}{2}, x = a$ and y = 0

Therefore ,equation of tangent is,

$y - y_1 = \frac{dy}{dx}(x - x_1), y - 0 = 0\,(x - a)$i.e y=0

27. Here,the given equations of lines are

$\frac{x}{-3} = \frac{y-2}{4} = \frac{z+1}{1}$ and $\frac{x-4}{1} = \frac{y-1}{-2} = \frac{z}{\frac{1}{2}}$, i.e., $\frac{x}{-3}=\frac{y-2}{4}=\frac{z+1}{1}$ and $\frac{x-4}{2}=\frac{y-1}{-4}=\frac{z}{1}$.

Therefore, the required plane passes through the point A (2, 0, -1) and it is parallel to the lines having direction ratios -3, 4, 1 and 2, -4, 1.

Here ($x_1$ = 2, $y_1$ = 0, $z_1$ = -1), ($a_1$ = -3 , $b_1$ = 4, $c_1$ = 1)

and ($a_2$ =2, $b_2$ = -4, $c_2$ = 1).

The required equation of the plane is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-0 & z+1 \\ -3 & 4 & 1 \\ 2 & -4 & 1 \end{vmatrix}=0$$

$\Rightarrow$ (x - 2)(4 + 4) - y(-3 - 2) + (z + 1)(12 - 8) = 0

$\Rightarrow$ 8(x - 2) + 5y + 4(z +1) = 0

$\Rightarrow$ 8x + 5y + 4z - 12 = 0

Therefore, the required Cartesian equation of the plane is 8x + 5y + 4z = 12 and its corresponding vector equation is $r\cdot(8\hat{i}+5\hat{j}+4\hat{k})=12$.

28. Let A and B denote respectively the events that first and second balls both drawn are white.

Therefore, we have to find P(A ∩ B)

Then, P(A) = P (white ball in the first draw) = $\frac{8}{12}$

After the occurrence of event A, we are left with 7 white and 4 red balls. The probability of drawing second white ball, given that the first ball drawn is white, is clearly the conditional probability of occurrence of B, given that A has occurred.Therefore,we have,

P(B/A) = $\frac{7}{11}$

By multiplication rule of probability, we have,

P(A ∩ B) = P(A) · P(B/A) = $\left(\frac{8}{12}\times\frac{7}{11}\right)=\frac{14}{33}$

OR

Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x = 0, 1, 2, .........n and q = (1 - p)

As the die is thrown 6 times the total number of outcomes will be $6^6$.

And we know that the favourable outcomes of getting at least 5 successes will be, either getting 2, 4 or 6 i.e, $\frac{1}{6}$ probability of each, total, $\frac{3}{6}$ probability, $p=\frac{3}{6},q=\frac{3}{6}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Therefore, the probability of getting at least 5 successes will be,

$=(^6C_5+^6C_6)(\frac{3}{6})^6$

$=\left(^6C_5+^6C_6\right)\frac{1}{64}$

$=\frac{7}{64}$

This is the required probability.

**Section - IV**

29. **Part I**: R = {($T_1$, $T_2$): $T_1$ is similar to $T_2$} and $T_1$, $T_2$ are triangles.

We know that each triangle similar to itself and thus ($T_1$, $T_1$)∈ R ∴ R is reflexive.

Also if two triangles are similar, then $T_1 \cong T_2 \Rightarrow T_1 \cong T_2$ ∴ R is symmetric.

Again, if $T_1 \cong T_2$ and $T_2 \cong T_3 \Rightarrow$ then $T_1 \cong T_3$ ∴ R is transitive.

Therefore, R is an equivalent relation.

**Part II**: It is given that $T_1$, $T_2$ and $T_3$ are right angled triangles.

$\Rightarrow$ $T_1$ with sides 3, 4, 5, $T_2$ with sides 5, 12, 13 and $T_3$ with sides 6, 8, 10

Since, two triangles are similar if corresponding sides are proportional.

Therefore, $\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{1}{2}$

Therefore, $T_1$ and $T_3$ are related.

30. According to the question, $y=sin^{-1}x$

Differentiating both sides w.r.t x,

$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\frac{dy}{dx}=1$

Squaring both sides,

$\Rightarrow (1-x^2)-\left(\frac{dy}{dx}\right)^2=1$

Differentiating both sides w.r.t x,

$\Rightarrow \left(1-x^2\right)2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2(-2x)=0$

Dividing both sides by $2\left(\frac{dy}{dx}\right)$,

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$

**Hence Proved.**

31. We have, xy = 4

$\Rightarrow y=\frac{4}{x}$

Differentiate it with respect to x,

$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{4}{x}\right)$

$\Rightarrow \frac{dy}{dx}=4\frac{d}{dx}\left(x^{-1}\right)$

$\Rightarrow \frac{dy}{dx}=4\left(-1\times x^{-1-1}\right)$

$\Rightarrow \frac{dy}{dx}=4\left(-\frac{1}{x^2}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{-4}{x^2}$

$\Rightarrow \frac{dy}{dx}=-\frac{4}{\left(\frac{4}{y}\right)^2}\left[\because x=\frac{4}{y}\right]$

$\Rightarrow \frac{dy}{dx}=-\frac{4y^2}{16}$

$\Rightarrow \frac{dy}{dx}=-\frac{y^2}{4}$

$\Rightarrow 4\frac{dy}{dx}=-y^2$

$\Rightarrow 4\frac{dy}{dx}=3y^2-4y^2$

$\Rightarrow 4\frac{dy}{dx}+4y^2=3y^2$

$\Rightarrow 4\left(\frac{dy}{dx}+y^2\right)=3y^2$

Divide both side by x,

$\Rightarrow \frac{4}{x}\left(\frac{dy}{dx}+y^2\right)=\frac{3y^2}{x}$

$\Rightarrow y\left(\frac{dy}{dx}+y^2\right)=\frac{3y^2}{x}$

$\Rightarrow x\left(\frac{dy}{dx}+y^2\right)=\frac{3y^2}{y}$

$\Rightarrow x\left(\frac{dy}{dx}+y^2\right)=3y$

LHS=RHS

Hence Proved.

OR

Here,

y = $x^2$ + x

since, y is a polynomial function, so it continuous differentiable,

Lagrange's mean value theorem is applicable, so, there exist a point c such that,

$\Rightarrow f'(c)=\frac{f(b)-f(a)}{b-a}$

$\Rightarrow 2c+1=\frac{f(1)-f(0)}{1-0}$

⇒ 2c + 1 = 2

$\Rightarrow c=\frac{1}{2}$

$\Rightarrow y=\left(\frac{1}{2}\right)^2+\frac{1}{2}$

$\Rightarrow y=\frac{3}{4}$

So, $(c,y)=\left(\frac{1}{2},\frac{3}{4}\right)$ is the required point according to the question .

32. Here, it is given the curve y = $x^2$ - x + 2 has a point crosses the y-axis.

The curve will be in the form of (0, y)

⇒ y = 0 - 0 + 2

⇒ y = 2

Therefore, the point at which curve crosses the y-axis (0, 2).

Now, differentiating the equation of curve w.r.t. x.

$\frac{dy}{dx}=x-1$

For (0, 2),

$\frac{dy}{dx}=-1$

Equation of the tangent:

(y - y1) = Slope of tangent × (x - $x_1$)

$\Rightarrow (y-2)=-1\times(x-0)$

⇒ y - 2 = x

⇒ x + y = 2

33. Let the given integral be,

$\mathrm{l}=\int\frac{2\tan x+3}{3\tan x+4}dx$

$=\int\left(\frac{\frac{2\sin x}{\cos x}+3}{\frac{3\sin x}{\cos x}+4}\right)dx$

$=\int\left(\frac{2\sin x+3\cos x}{3\sin x+4\cos x}\right)dx$

Let (2 sin x + 3 cos x) = A(3 sin x + 4 cos x) + B (3 cos x - 4 sin x) ...(i)

⇒ 2 sin x + 3 cos x = (3A - 4B) sin x + (4A + 3B) cos x

Equating the coefficients of like terms

3A - 4B = 2 ...(ii)

4A + 3B = 3 ...(iii)

Multiplying equation (ii) by 3 and equation (iii) by 4, then by adding them we get

25A = 18

$\Rightarrow A=\frac{18}{25}$

Putting the value of A in equation (ii) we get,

$\Rightarrow B=\frac{1}{25}$

Thus, by substituting the values of A and B in equation (i), we get

$I=\int\left\{\frac{\frac{18}{25}(3\sin x+4\cos x)+\frac{1}{25}(3\cos x-4\sin x)}{3\sin x+4\cos x}\right\}dx$

$=\frac{18}{25}\int dx+\frac{1}{25}\int\left(\frac{3\cos x-4\sin x}{3\sin x+4\cos x}\right)dx$

Putting 3 sin x + 4 cos x = t

(3 cos x - 4 sin x) dx = dt

$\therefore I=\frac{18}{25}x+\frac{1}{25}\int\frac{1}{t}dt$

$=\frac{18x}{25}+\frac{1}{25}$ In |t| + C

$=\frac{18x}{25}+\frac{1}{25}$ In |3 sin x + 4 cos x| + C

34. Here, the given curves are:

y = $x^2$ .... (i)

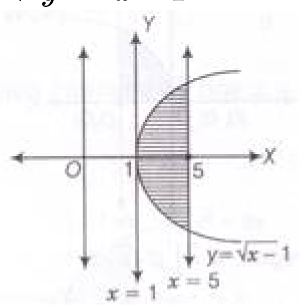
y = x ..... (ii)

Using eqn. (ii) in (i), gives

or $x^2 - x = 0$
or $x.(x - 1) = 0$
$\Rightarrow x = 0, 1.$
But $y = x \Rightarrow y = 0, 1$
Thus, (0, 0) and (1, 1) are the points of intersection.
Area between ($y = x$ and $y = x^2$) is:
$A = \int \left|\left(x - x^2\right) \cdot dx\right|$
$= \left|\frac{(x^2)}{2} - \frac{(x^3)}{3}\right|$
Using limits from 0, to 1, we get
$A = \left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0)$
$= \frac{1}{6}$ sq.unit

OR

Given equation of the curve is $y = \sqrt{x-1}$
$\Rightarrow y^2 = x - 1$




$\therefore$ Area of shaded region, $A = \int_1^5 (x-1)^{1/2} dx = \left[\frac{2.(x-1)^{3/2}}{3}\right]_1^5$
$= \left[\frac{2}{3}.(5-1)^{3/2} - 0\right] = \frac{16}{3}$ sq unit

35. We can write the given differential equation as,
$\left(\frac{y}{x} + y^2 \sin \frac{y}{x}\right) dx = \left(xy \sin \frac{y}{x} - x^2 \cos \frac{y}{x}\right) dy$
$\Rightarrow \frac{dy}{dx} = \frac{\left\{xy \cos \frac{y}{x} + y^2 \sin \frac{y}{x}\right\}}{\left\{xy \sin \frac{y}{x} - x^2 \cos \frac{y}{x}\right\}}$
$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right) \cos\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 \sin\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right) \sin\left(\frac{y}{x}\right) - \cos\left(\frac{y}{x}\right)} = f\left(\frac{y}{x}\right)$ ...(i)
Therefore, the given differential equation is homogeneous.
Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i),
$\Rightarrow \quad x\frac{dv}{dx} = \left\{\frac{(v \cos v + v^2 \sin v)}{(v \sin v - \cos v)} - v\right\}$
$\Rightarrow \quad x\frac{dv}{dx} = \frac{2v \cos v}{(v \sin v - \cos v)}$
$\Rightarrow \int \frac{(v \sin v - \cos v)}{v \cos v} dv = \int \frac{2}{x} dx$
$\Rightarrow \int \tan v \, dv - \int \frac{dv}{v} = \int \frac{2}{x} dx$
$\Rightarrow -\log|\cos v| - \log|v| - 2\log|x| = \text{constant}$
$\Rightarrow \log|\cos v| + \log|v| + 2\log|x| = \log|C_1|$ where $C_1$ is an arbitrary constant
$\Rightarrow \log|x^2 v \cos v| = \log|C_1|$
$\Rightarrow x^2 v \cos v = \pm C_1 = C$(say)
$\Rightarrow x y \cos \frac{y}{x} = C$, which is the required solution $\left[\because v = \frac{y}{x}\right]$

**Section - V**

36. Let $E_1$: Transferred ball is green
$E_2$: Transferred ball is red
A: Green ball is found

Here, $P(E_1) = \frac{2}{6}$, $P(E_2) = \frac{4}{6}$

$P(A/E_1) = \frac{6}{9}$, $P(A/E_2) = \frac{5}{9}$

Using Baye's theorem

$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)}$

$= \frac{\frac{2}{6}\times\frac{6}{9}}{\frac{2}{6}\times\frac{6}{9}+\frac{4}{6}\times\frac{5}{9}}$

$= \frac{12}{12+20} = \frac{3}{8}$

OR

Let us define the events as

$E_1$: Boy is selected

$E_2$: Girl is selected

A: The student has an IQ of more than 150 students.

Now, $P(E_1)=60\% = \frac{60}{100}$

Now, $P(E_2 = 40\% = \frac{40}{100}$) [∵ in the class 60% students are boys, so 40% are girls, given]

Now, $P(A/E_1)$ = Probability that boys has an IQ of

more than 150

$= 5\% = \frac{5}{100}$

and $P(A/E_2)$ = Probability that girls has an IQ

of more than $150 = 10\% = \frac{10}{100}$

The probability that the selected boy having IQ more than 150 is

$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$

[by Baye's theorem]

$= \frac{\frac{60}{100}\times\frac{5}{100}}{\left(\frac{60}{100}\times\frac{5}{100}\right)+\left(\frac{40}{400}\times\frac{10}{100}\right)}$

$= \frac{300}{300+400} = \frac{300}{700} = \frac{3}{7}$

Hence, the required probability is 3/7

37. Now, given equation of lines are

$\vec{r} = (\hat{i}+\hat{j}) + \lambda(\hat{i}+2\hat{j}-\hat{k})$

and $\vec{r} = (\hat{i}+\hat{j}) + \mu(\hat{-i}+\hat{j}-2\hat{k})$

On comparing these equations with standard equation of line, $\vec{r} = \vec{a} + \lambda\vec{b}$, we get

$\overrightarrow{a_1} = \hat{i}+\hat{j}, \overrightarrow{a_2} = \hat{i}+\hat{j}, \overrightarrow{b_1} = \hat{i}+2\hat{j}-\hat{k}$and

$\overrightarrow{b_2} = -\hat{i}+\hat{j}-2\hat{k}$

Now, the equation of plane containing both the lines is given by

$(\vec{r}-\vec{a}_1)\cdot\vec{n} = 0$

where, $\vec{n}$ is normal to both the lines.

Clearly, $\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$

$= \hat{i}(-4+1) - \hat{j}(-2-1) + \hat{k}(1+2)$

$= -3\hat{i}+3\hat{j}+3\hat{k}$

The required equation of plane is

$[\overrightarrow{r} - (\hat{i}+\hat{j})]\cdot(-3\hat{i}+3\hat{j}+3\hat{k}) = 0$

$\Rightarrow \vec{r}\cdot(-3\hat{i}+3\hat{j}+3\hat{k}) = (\hat{i}+\hat{j})\cdot(-3\hat{i}+3\hat{j}+3\hat{k})$

$\Rightarrow \vec{r}\cdot(-3\hat{i}+3\hat{j}+3\hat{k}) = -3+3 = 0$

$\Rightarrow \quad \vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k}) = 0$

Now, the length of perpendicular from the point (2, 1, 4) to the above plane

$= \frac{|(2\hat{i}+\hat{j}+4\hat{k})\cdot(-\hat{i}+\hat{j}+\hat{k})|}{\sqrt{(-1)^2+1^2+1^2}}$

$= \frac{|-2+1+4|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}} = \sqrt{3}.units$

OR

Given points are A (3, -4, -5), B (2, -3, 1)

Direction ratios of line AB are (3 - 2, -4 + 3, -5 - 1)

i.e (1, -1, -6)

Eq. of line AB is,

$\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = \lambda$

$\Rightarrow x = \lambda + 2, y = -\lambda - 3, z = -6\lambda + 1$

Thus, any point on the line is $(\lambda + 2, -\lambda - 3, -6\lambda + 1)$

This point lies on the plane 2x + y + z = 7t

Therefore $2(\lambda + 2) - \lambda - 3 - 6\lambda + 1 = 7$

$\Rightarrow -5\lambda + 2 = 7$

$\Rightarrow -5\lambda = 5$

$\Rightarrow \lambda = -1$

Required point of intersection of line and plane is

(-1 + 2, 1 - 3, 6 + 1)

(1, - 2, 7)

38. To Maximize Z = 4x + y ......(i)

subject to the constraints:

x + y $\leq$ 50 .....(ii)

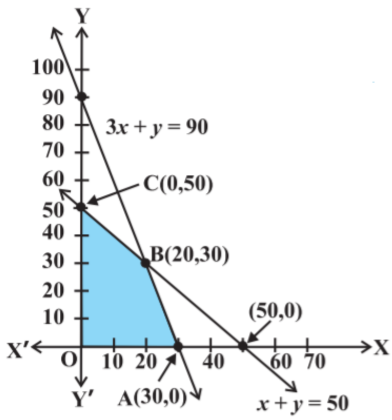
3x + y $\leq$ 90 ......(iii)

x $\geq$ 0, y $\geq$ 0 .....(iv)

The shaded region in a figure is the feasible region determined by the system of constraints (ii) to (iv). We observe that the feasible region OABC is bounded. So, we now use Corner Point Method to determine the maximum value of Z.

The coordinates of the corner points O, A, B and C are (0, 0), (30, 0), (20, 30) and (0, 50) respectively. Now we evaluate Z at each corner point.

| Corner Point | Corresponding value of Z |
|---|---|
| (0, 0) | 0 |
| (30, 0) | 120 |
| (20, 30) | 110 |
| (0, 50) | 50 |




Hence, maximum value of Z is 120 at the point (30, 0).

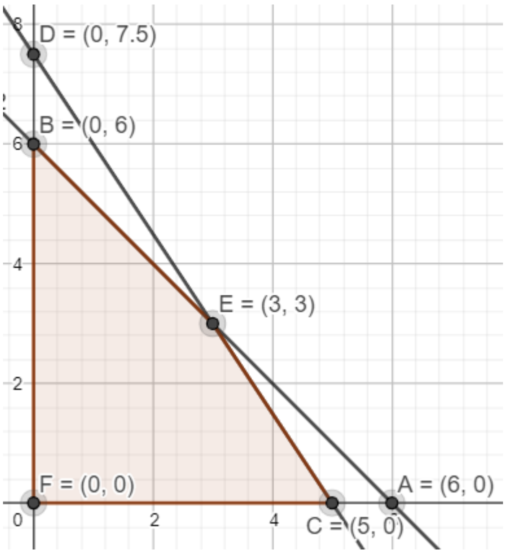
OR

Let x trunks of first type and y trunks of second type were manufactured. Number of trunks cannot be negative. Therefore,

x, y $\geq$ 0
According to the question, the given information can be tabulated as

| | Machine A (hrs) | Machine B (hrs) |
|---|---|---|
| First type(x) | 3 | 3 |
| Second type(y) | 3 | 2 |
| Availability | 18 | 15 |

Therefore, the constraints are
3x + 3y $\leq$ 18
3x + 2y $\leq$ 15
He earns a profit of ₹30 and ₹25 per trunk of the first type and the second type respectively. Therefore, profit gained by him from x trunks of first type and y trunks of second type is ₹30x and ₹25y respectively.
Total profit = z = 30x + 25y
which is to be maximised Thus, the mathematical formulation of the given linear programmimg problem is
Max z = 30x + 25y
subject to
3x + 3y $\leq$ 18
3x + 2y $\leq$ 15
x, y $\geq$ 0
First we will convert inequations into equations as follows:
3x + 3y = 18, 3x + 2y = 15, x = 0 and y = 0
Region represented by 3x + 3y $\leq$ 18 The line 3x + 3y = 18 meets the coordinate axes at $A_1$(6, 0) and $B_1$(0, 6) respectively. By joining these points we obtain the line 3x + 3y = 18. Clearly (0, 0) satisfies the 3x + 3y = 18 . So, the region which contains the origin represents the solution set of the inequation 3x + 3y $\leq$ 18
Region represented by 3x + 2y $\leq$ 15 :
The line 3x + 2y = 15 meets the coordinate axes at $C_1$(5, 0) and $D_1\left(0, \frac{15}{2}\right)$ respectively. By joining these points we obtain the line
3x + 2y = 15 . Clearly (o, 0) satisfies the inequation 3x + 2y $\leq$ 15 . So, the region which contains the origin represents the solution set of the inequation 3x + 2y $\leq$ 15
Region represented by x $\geq$ 0 and y $\geq$ 0 :
since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0
The feasible region determined by the system of constraints 3x + 3y $\leq$ 18, 3x + 2y $\leq$ 15, x $\geq$ 0 and y $\geq$ 0 are as follows.




The corner points are O(0, 0), $B_1$(0, 6), $E_1$(3, 3) and $C_1$(5, 0)
The values of Z at these corner points are as follows
Corner point : Z = 30x + 25y
O : 0

$B_1$ : 150

$E_1$ : 165

$C_1$ : 150

The maximum value of Z is 165 which is attained at $E_1(3, 3)$

Thus, the maximum profit is ₹165 obtained when 3 units of each type of trunk is manufactured.